1. (Problem 1.7 in Attix) A point source, isotropically emitting $10^8$ fast neutrons ($n_o$) per second, falls out of its shield onto a railroad platform, 3 meters (horizontally) from the track. A train goes by at 60 mi h$^{-1}$. Ignoring scatter and attenuation, what is the fluence of neutrons that would strike a passenger at the same height above the track as the source?

The fluence $\Phi$ is given by the integral $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi \, dt \, dt$. The fluence rate is the number of particles per unit area per unit time, or

$$\frac{d\Phi}{dt} = \frac{1}{4\pi^2} \frac{dN}{dt}.$$  

We can write

$$dt = \frac{dx}{v},$$  

where $v$ is the speed of the train, and $r$, the distance from the source to a point on the train is given by

$$r^2 = x^2 + h^2.$$  

Putting it all together, we can write

$$\Phi = \int_{x=-\infty}^{x=+\infty} \frac{1}{4\pi(x^2 + h^2)} \frac{dN}{dt} \frac{dx}{v}.$$  

Making use of the integral$^1$

$$\int \frac{dx}{(x^2 + h^2)} = \frac{1}{h} \arctan\left( \frac{x}{h} \right),$$

we write

$$\Phi = \frac{1}{4\pi v h} \frac{dN}{dt} \arctan\left( \frac{x}{h} \right) \bigg|_{-\infty}^{\infty}.$$  

Evaluating the arctan function between the two endpoints gives us a factor $\pi$, so the fluence is given by

$$\Phi = \frac{1}{4vh} \frac{dN}{dt}.$$  

$^1$ See, for example the URL, [http://integrals.wolfram.com/index.jsp](http://integrals.wolfram.com/index.jsp)
We have \( \frac{dN}{dt} = 10^8 \text{ sec}^{-1} \), \( v = 60 \text{ mi h}^{-1} \), and \( h = 3 \text{ m} \), so

\[
\Phi = \frac{\text{hr}}{4 \cdot 60 \text{ mi} \cdot 3 \text{ m}} \cdot 10^8 \text{ sec}^{-1}
\]

\[
= \frac{10^8 \text{ hr} \cdot 60 \text{ min} \cdot 60 \text{ sec} \cdot \text{mi} \cdot \text{ft}}{4 \cdot 60 \text{ mi} \cdot 3 \text{ m} \cdot \text{sec} \cdot \text{hr} \cdot \text{min} \cdot 5280 \text{ ft} \cdot 0.3048 \text{ m}}
\]

\[
= 3.11 \times 10^5 \text{ m}^2
\]

2. Show that absorbed dose has the same units as velocity squared. What, if anything, does this imply about the relation of dose to velocity?

Units for absorbed dose are \([\text{energy mass}^{-1}]\). Because the units for energy are \([\text{mass velocity}^2]\), the units for absorbed dose are \([\text{velocity}^2]\). This implied nothing about the relation of dose to velocity because dose and velocity are two completely different quantities; they just have the same units.

3. (Problem 1-5 in Attix) A point source of \(^{60}\text{Co}\) emits equal numbers of photons of 1.17 and 1.33 MeV giving a flux density (ICRU #60 calls it “fluence rate”) of \(5.7 \times 10^9\) photon cm\(^{-2}\) s\(^{-1}\) at a specified location. What is the energy flux density (ICRU # 60 now calls it “energy fluence rate”) expressed in MeV cm\(^{-2}\) s\(^{-1}\) and in J m\(^{-2}\) min\(^{-1}\)?

The energy fluence rate \(\Psi\) can be obtained from the photon fluence rate \(\Phi\) by multiplying the photon fluence rate by the energy. When we have a polyenergetic spectrum, we can sum over the energy components.

\[
\Psi = \sum_i f(E_i)E_i \Phi(E_i)
\]

where \(f(E_i)\) is the fraction of photons with energy \(E_i\). In this example, the fraction of photons with each of the two energies is 0.5. Thus

\[
\Psi = (0.5 \cdot 1.17 \cdot 5.7 \times 10^9 + 0.5 \cdot 1.33 \cdot 5.7 \times 10^9) \text{ MeV cm}^2 \text{ sec}^{-1}
\]

\[
= 7.13 \times 10^9 \text{ MeV cm}^2 \text{ sec}^{-1}
\]

\[
= 7.13 \times 10^9 \frac{\text{MeV}}{\text{cm}^2 \text{ sec}} \cdot \frac{10^6 \text{ eV}}{\text{MeV}} \cdot \frac{1.6 \times 10^{-19} \text{ J}}{10^4 \text{ cm}^2} \cdot \frac{1 \text{ sec}}{60 \text{ min}}
\]

\[
= 684 \text{ J m}^2 \text{ min}^{-1}
\]
4. (Problem 1-8 in Attix) An x-ray field at a point P contains $7.5 \times 10^8$ m$^{-2}$ s$^{-1}$ keV$^{-1}$, uniformly distributed from 10 to 100 keV. (We might symbolize this, using ICRU#60 notation, as \( \Phi_E = \frac{d\Phi}{dE} = \frac{d}{dE} \left( \frac{d\Phi}{dt} \right) = \frac{d}{dt} \left[ \frac{dN}{da} \right] \)).

a. What is the photon fluence rate at P?

The photon fluence rate is found by integrating \( \Phi_E \) over the energy spectrum of the photons, so we write

\[
\Phi = \int_{10 \text{ keV}}^{100 \text{ keV}} \Phi_E dE
\]

\[
= 90 \text{ keV} \cdot 7.5 \times 10^8 \text{ m}^{-2} \text{ sec}^{-1} \text{ keV}^{-1}
\]

\[
= 6.75 \times 10^{10} \text{ m}^{-2} \text{ sec}^{-1}
\]

b. What would be the photon fluence in one hour?

Because the photon fluence rate is constant the photon fluence is found by multiplying the photon fluence rate by the time, so we write

\[
\Phi = \Phi t
\]

\[
= 6.75 \times 10^{10} \text{ m}^{-2} \text{ sec}^{-1} \cdot 1 \text{ hr}
\]

\[
= 6.75 \times 10^{10} \frac{\text{hr} \times 3600 \text{ sec}}{\text{m}^2 \cdot \text{sec} \cdot \text{hr}}
\]

\[
= 2.43 \times 10^{14} \text{ m}^{-2}
\]

c. What is the corresponding energy fluence in J m$^{-2}$?

The energy fluence is determined by first integrating \( E \Phi_E \) over the energy spectrum of the photons to get the energy fluence rate.

\[
\Psi = \int_{10 \text{ keV}}^{100 \text{ keV}} E\Phi_E dE
\]

\[
= \frac{1}{2} \Phi_E E^2 \bigg|_{10 \text{ keV}}^{100 \text{ keV}}
\]

\[
= 0.5 \cdot 7.5 \times 10^8 \text{ m}^{-2} \text{ sec}^{-1} \text{ keV}^{-1} \cdot (100^2 \text{ keV}^2 - 10^2 \text{ keV}^2)
\]

\[
= 3.71 \times 10^{12} \text{ keV m}^{-2} \text{ sec}^{-1}
\]
We now multiply the energy fluence rate by the time to get the energy fluence.

\[ \Psi = \Psi t \]

\[ = 3.71 \times 10^{12} \text{keV m}^{-2} \text{sec}^{-1} \cdot \text{hr} \]

\[ = 3.71 \times 10^{12} \frac{\text{keV} \cdot \text{hr} \times 3600 \text{sec}}{\text{m}^2 \cdot \text{sec} \cdot \text{hr}} \]

\[ = 1.34 \times 10^{16} \text{keV m}^{-2} \]

\[ = 1.34 \times 10^{16} \frac{\text{keVm}^{-2} \cdot 1.6 \times 10^{-19} \text{J eV}^{-1} \cdot 10^3 \text{eV keV}^{-1}}{1 \text{sec} \cdot \text{hr}} \]

\[ = 2.14 \text{J m}^{-2} \]