Radiation Quantities and Units

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Lecture Objectives

• Define and identify units for the following:
  – Exposure
  – Kerma
  – Absorbed dose
  – Dose equivalent
  – Relative biological effectiveness
  – Activity

• Define and identify units for the following:
  – Particle number
  – Radiation energy
  – Particle flux
  – Energy flux
  – Particle fluence
  – Energy fluence
  – Planar fluence
Lecture Objectives

• Define and identify units for the following:
  – Cross section
  – Linear attenuation coefficient
  – Mass attenuation coefficient
  – Mass stopping power

• Note: This lecture is an introduction to radiation quantities and units. This topic will be presented in significantly more depth later in the course

First things first

• What are we talking about in this course?
  – Ionizing radiation: Sufficient energy to excite and ionize atoms of matter

Types of ionizing radiation

• Gamma rays
  – Electromagnetic radiation emitted as a result of nuclear interactions
    • Changes in nuclear energy levels
    • Annihilation of positrons
    • Energy range: some keV to a few MeV
Types of ionizing radiation

• X-rays
  – Electromagnetic radiation emitted as a result of electronic interactions
    • Changes in electronic energy levels – characteristic x-rays
    • Deceleration of charged particles (usually electrons) – Bremsstrahlung ("braking radiation")
  • Energy ranges:
    – 0.1-20 kV Grenz rays
    – 20-120 kV diagnostic x-rays
    – 120-300 kV orthovoltage x-rays
    – 300 kV-1 MV intermediate-energy x-rays
    – > 1 MV megavoltage x-rays

Types of ionizing radiation

• Electrons
  – Charged particles emitted from a nucleus – β rays (particles)
  – Fast electrons resulting from charged particle collision – δ rays
  – Continuous accelerated beams
    • X-ray tube
    • Van de Graaff generator
  – Pulsed accelerated beams
    • Linear accelerator (Linac)
    • Betatron
    • Microtron

Types of ionizing radiation

• Heavy charged particles
  – Protons
  – Deuterons
  – Alpha particles
  – Heavy atom nuclei
  – Pions

• Neutrons
  – Obtained from nuclear interactions involving high-energy charged particles or photons
Types of ionizing radiation

• Directly ionizing radiation
  – Fast charged particles
  – Deliver energy to matter directly
  – Coulomb interactions
• Indirectly ionizing radiation
  – X-rays, γ-rays, neutrons
  – Transfer energy to charged particles
  – Secondary charged particles deliver energy to matter

Exposure

• Definition – Exposure is the absolute value of the total charge of ions of one sign produced in a small mass of air, when all electrons liberated by photons in air are completely stopped in air, divided by the mass of air

\[ X = \frac{dQ}{dm} \]

Some clarification needed

• “absolute value of the total charge of ions of one sign”
  – Radiation causes ionization
  – Total charge produced is zero (positive balances out negative)
  – Consequently, we only look at charge of one sign or the other
Some clarification needed

• “produced in a small mass of air”
  – Ionization is a stochastic process
  – Need to have a large enough sample to determine a meaningful expectation value of charge production

The devil in the details

• “all electrons liberated by photons in air are completely stopped in air”
  – Aren’t we measuring photon exposure?
  – What do electrons have to do with this?
  – How does this make things complicated?

Photon interactions

• Photons interacting with absorber (air molecules) give rise to secondary radiations (electrons) which, in turn, interact further with absorber
  – Single ionization (due to photon) yields many ionizations (due to electrons) downstream
Path length of electrons produced by photons

<table>
<thead>
<tr>
<th>Photon Energy (MeV)</th>
<th>Maximum Electron Path Length in Air (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>1.0</td>
<td>3.0</td>
</tr>
<tr>
<td>3.0</td>
<td>12.2</td>
</tr>
<tr>
<td>10.0</td>
<td>40.9</td>
</tr>
</tbody>
</table>

Photon interactions

- Photons interacting with absorber (air molecules) give rise to secondary radiations (electrons) which, in turn, interact further with absorber
  - Not possible to track individual electrons producing ionizations downstream
  - Introduces concept of electronic (charged particle) equilibrium

Charged Particle Equilibrium

- Energy deposited by charged particles produced inside a volume and deposited outside the volume is equal to energy deposited by charged particles produced outside the volume and deposited inside the volume
Charged Particle Equilibrium

- Working definition – number and energy spectrum of charged particles constant within volume
- Major violations
  - Near radiation source
  - Near material interfaces

Exposure

- Definition – Exposure is the absolute value of the total charge of ions of one sign produced in a small mass of air, when all electrons liberated by photons in air are completely stopped in air, divided by the mass of air
  \[ X = \frac{dQ}{dm} \]

Limitations of Exposure

- Must occur in air
- Defined only for photons (x rays, gamma rays)
- Not defined for energies > 3 MeV
  - Need for charged particle equilibrium
  - Need volume > 3-4 m for high energies
Units of Exposure

- Units of charge per unit mass
  - C kg\(^{-1}\)
  - No special unit for exposure
- Old unit – Roentgen
  - \(1 \text{ R} = 2.58 \times 10^{-4} \text{ C kg}^{-1}\)
- Can outlaw unit, but cannot outlaw quantity! – W Hanson

Kerma

- Kinetic Energy Released in Matter
  \[
  K = \frac{dE_k}{dm}
  \]
- \(dE_k\) – sum of initial kinetic energies of all charged particles liberated by uncharged ionizing particles in material of mass \(dm\)

Kerma

- Incident photon interacts with matter
  - Some energy may be transferred to charged particles
  - Some energy may be transferred to scattered photon
- Kerma looks only at that energy transferred to charged particles
Kerma

• Convenience – ionizing events that follow primary interactions need not be considered
  – Spatial distribution of charged particles can be ignored

Units of Kerma

• Units of energy per unit mass
  – J kg\(^{-1}\)
  – Special unit – Gray
    • 1 Gy – 1 J kg\(^{-1}\)

Limitations of Kerma

• None
  – Defined in all materials
  – Defined for all uncharged ionizing radiations (x rays, gamma rays, neutrons)
  – Defined at all energies
  – Can be measured any way you want
Determination of Air Kerma

• Measure exposure
• Multiply by energy transferred to medium per ionization
• Make units make sense – conversion factors
• May need to correct for re-radiation (negligible at high energies)
  – We’ll learn about that in electron interactions

Example of Measurement

• What is air kerma corresponding to 1 R exposure?
• Measure exposure – 1 R = 2.58 × 10⁻⁴ C kg⁻¹
• Convert C to ion pairs (IP)

\[
1 \text{R} = 2.58 \times 10^{-4} \frac{\text{C}}{\text{kg}} \times 1.6 \times 10^{19} \frac{\text{IP}}{\text{C}} = 4.128 \times 10^{15} \frac{\text{IP}}{\text{kg}}
\]

Example of Measurement

• Multiply by energy transferred to medium per ionization – 33.7 eV/IP

\[
1 \text{R} = 4.128 \times 10^{15} \frac{\text{IP}}{\text{kg}} \times 33.7 \frac{\text{eV}}{\text{IP}} = 139.1 \times 10^{15} \frac{\text{eV}}{\text{kg}}
\]
Example of Measurement

- Divide by $1.6 \times 10^{19}$ eV/J

\[
1 \text{R} = 139.1 \times 10^{15} \text{ eV/kg} \times \frac{1}{1.6 \times 10^{19} \text{ eV}} \\
= 86.9 \times 10^{-4} \frac{\text{J}}{\text{kg}} \\
= 0.869 \text{ cGy}
\]

Example of Measurement

- Air kerma corresponding to 1 R is 0.869 cGy.

Cema

- Rarely used
- Kerma equivalent for electrons
- Charged particle Energy imparted to Matter

\[
c = \frac{dE_c}{dm}
\]

- $dE_c$ – energy lost by charged particles in electronic collisions including the energy expended against binding energies and any kinetic energy of the liberated electrons (secondary electrons)
Units of Cema

• Units of energy per unit mass
  – J kg\(^{-1}\)
  – Special unit – Gray
    • 1 Gy – 1 J kg\(^{-1}\)

Similarities to Kerma

• Count only energy lost at time of collision
  – We do not care how this energy is ultimately expended in the medium

Difference from Kerma

• Cema accounts for binding energy of electrons
  – Most photon interactions are high-energy interactions so binding energy usually insignificant
  – Electron interactions are low-energy so binding energy is significant
Summary

- Kerma – energy imparted to medium by uncharged particles
- Cema – energy lost by charged particles

Absorbed Dose

\[ D = \frac{dE}{dm} \]

- \( dE \) – mean energy imparted by ionizing radiation to matter of mass \( dm \)
- Dose includes secondary radiation

Units of Dose

- Units of energy per unit mass
  - J kg\(^{-1}\)
  - Special unit – Gray
    - 1 Gy – 1 J kg\(^{-1}\)
Limitations of Dose

- None
  - Defined in all materials
  - Defined for all ionizing radiations
  - Defined at all energies

Dose Equivalent

- Not all ionizing radiations have the same biological effect
- Used for radiation protection purposes only
  \[ H = D \times Q \times N \]
  - \( D \) = physical dose
  - \( Q \) = quality factor that weights dose for biological effectiveness
  - \( N \) = product of all other relevant weighting factors (typically 1)

Units of Dose Equivalent

- Units of energy per unit mass – weighted
  - J kg\(^{-1}\)
  - Special unit – Sievert
    - 1 Sv = 1 J kg\(^{-1}\)
Relative Biological Effectiveness - RBE

- Used in radiobiology and radiation oncology
- Accounts for differences in biological effect among radiations
  \[ \text{RBE dose} = D \times \text{RBE} \]
- Specific to radiation spectrum, organ of interest, end point of interest (cell death, tumor control)

Relative Biological Effectiveness - RBE

- Encounter RBE in proton radiation therapy
- Precise RBE of protons not known – taken to be \( \sim 1.1 \)
- Proton doses expressed as “Cobalt Gray equivalents” = \( 1.1 \times \text{dose} \)

Activity

- Amount of radioactive element in a particular energy state at a given time that will decay to another state in a given time interval
  \[ A = \frac{dN}{dt} \]
- \( dN \) – expectation value of the number of spontaneous nuclear transitions from a given excited state of an isotope in a time \( dt \)
Activity

- Activity represents source decay rate only
- Net value of $dN/dt$ may be affected by
  - Production of radioisotope nuclei
  - Alternate disappearance mechanisms, e.g. biological removal
- Activity does not represent emission rate of radiation produced in decay
  - Given radiation may be emitted in only fraction of decays

Units of Activity

- Units of number (dimensionless) per unit time
  - s$^{-1}$
  - Special unit – Becquerel
    - 1 Bq – 1 s$^{-1}$
  - Old unit – Curie
    - 1 Ci – $3.7 \times 10^{10}$ s$^{-1}$

Specific Activity

- Activity per unit mass of radioisotope
- For pure ("carrier-free") sample

\[
specific\ activity = \frac{\lambda N}{NM/A} = \frac{\lambda}{M}
\]

- $M$ = molecular weight
- $A$ = Avogadro's number
Radiometric Quantities

• Particle Number
  – \( N = \) number of particles emitted, transferred, received, etc
  – Unit: dimensionless

• Radiant Energy
  – \( R = NE \) where \( E \) is energy (excluding rest mass) of particle emitted, transferred, received
  – Unit: energy [J]

Radiometric Quantities

• Particle Flux
  \[ \dot{N} = \frac{dN}{dt} \]
  – \( dN = \) increment of particle number per unit time \( dt \)
  – Unit: number per unit time \([s^{-1}]\)

Radiometric Quantities

• Energy Flux
  \[ \dot{R} = \frac{dR}{dt} \]
  – \( dR = \) increment of radiation energy per unit time \( dt \)
  – Unit: energy per unit time \([J \ s^{-1}]\)
Radiometric Quantities

- **(Particle) Fluence**
  \[ \Phi = \frac{dN}{da} \]
  - \( dN \): number of particles incident on sphere of cross-sectional area \( da \)
  - Unit: number per unit area \([\text{m}^{-2}]\)

Comment on Fluence

- We need to have radiation interact in some volume or pass through some cross-sectional area. We define a cross-sectional area so that the beam is always perpendicular to a great circle of area \( da \).

Radiometric Quantities

- **Energy Fluence**
  \[ \Psi = \frac{dR}{da} \]
  - \( dR \): energy incident on sphere of cross-sectional area \( da \)
  - Unit: energy per unit area \([\text{J m}^{-2}]\)
Radiometric Quantities

- **(Particle) Fluence Rate**
  \[ \Phi = \frac{d\Phi}{dt} \]
  - Unit: number per unit area per unit time
  \[ [\text{m}^{-2} \text{s}^{-1}] \]

Radiometric Quantities

- **Energy Fluence Rate**
  \[ \Psi = \frac{d\Psi}{dt} \]
  - Unit: energy per unit area per unit time
  \[ [\text{J m}^{-2} \text{s}^{-1}] \]

Note

- Energy fluence has been called flux, particularly in engineering. Fluence rate has been called particle flux density. ICRU 60 discourages this use.
Planar Fluence

- Number of particles crossing a fixed plane in either direction per unit area of the plane

Planar Fluence

- The planar cross-section retains the same planar fluence, which suggests that the fluence of the total beam has not increased.
- However, the cross-section of the sphere reflects the increase in the fluence through a given $\Delta m$, which results in an increase in the dose rate.

Planar Fluence

- Effect sometimes observed in broad-beam geometry
  - Planar fluence behind attenuating layer can be greater than planar fluence incident on layer
  - Energy imparted into detector greater behind attenuating layer provided radiation penetrates detector
Interaction Coefficients

• Cross Section
  \[ \sigma = \frac{P}{\Phi} \]
  – Probability of an interaction for a given target when bombarded with a given particle fluence
  – Interaction – an event that changes the energy or direction of the incident radiation

• Cross Section
  \[ \sigma = \frac{P}{\Phi} \]
  – Unit: probability per unit fluence = area [m²]
  – Special unit: barn – 1 barn = 10⁻²⁴ cm²

Interaction Coefficients

• Linear attenuation coefficient
  \[ \mu = \frac{dN}{N} \]
  – Fraction of particles that interact in a given path length dl
  – Unit: fraction per unit length [m⁻¹]
Interaction Coefficients

- Mass attenuation coefficient - $\mu/\rho$
- Mass attenuation coefficient – fraction of particles that interact in density-weighted path length
- Note that cross section [m$^2$] multiplied by density-weighted path length [kg m$^{-2}$] gives mass, which is $dm$ in definition of dose

Interaction Coefficients

- Stopping power
  \[ S = \frac{dE}{dl} \]
  - Energy loss per unit path length $dl$
  - Unit: energy per unit length [J m$^{-1}$]
  - Mass stopping power - $S/\rho$

Interaction Coefficients

<table>
<thead>
<tr>
<th>Mass attenuation coefficient</th>
<th>Mass stopping power</th>
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<tbody>
<tr>
<td>Used to describe photon interactions</td>
<td>Used to describe charged-particle interactions</td>
</tr>
<tr>
<td>Fraction of particles per density-weighted path length</td>
<td>Energy loss per density-weighted path length</td>
</tr>
</tbody>
</table>