1. In the radioactive decay of $^{226}$Ra, how much kinetic energy is imparted to the recoil nucleus? (Hint: Recall your basic conservation laws from freshman physics.)

Let $Q$ be the transition energy of the decay process ($Q=4.78$ MeV). Let $M$ and $V$ be the mass and velocity of the recoil nucleus, and $m$ and $v$ be the mass and velocity of the alpha particle. Then, from freshman physics, we know that the transition energy goes into kinetic energy of both the recoil nucleus and the alpha particle. Thus

$$Q = \frac{1}{2} MV^2 + \frac{1}{2} mv^2.$$

The other equation we need is the conservation of momentum,

$$MV = mv,$$

or,

$$V = \frac{mv}{M}.$$

Substituting this value for $V$ in the energy conservation equation, we have

$$Q = \frac{1}{2} M \frac{m^2v^2}{M^2} + \frac{1}{2} mv^2$$

$$= \frac{1}{2} mv^2 \left( \frac{m}{M} + 1 \right),$$

$$= E \left( \frac{m}{M} + 1 \right),$$

where $E$ is the kinetic energy of the alpha particle. Solving for $E$, we have

$$E = \frac{Q}{1 + \frac{m}{M}}.$$

The kinetic energy of the recoil nucleus is then
The ratio of the mass of an alpha particle to that of a $^{222}$Rn nucleus is approximately $4/222 = 0.0180$, so that the energy imparted to the recoil nucleus is given by

$$Q - E = Q \left(1 - \frac{1}{1 + \frac{m}{M}}\right)$$

$$= Q \left(\frac{1 + \frac{m}{M} - 1}{1 + \frac{m}{M}}\right)$$

$$= Q \frac{m}{M} \left(1 + \frac{m}{M}\right)^{-1}$$

$$Q - E = 4.78 \times 0.0180 (1.018)^{-1} \text{ MeV}$$

$$= 8.46 \times 10^{-2} \text{ MeV}$$

2. (J & C 3-2) Radium has a half-life of 1620 yr. A radium source contains 1 mg of radium. The mass number is 226, and Avogadro’s number is $6.02 \times 10^{23}$. Find the transformation constant for radium and the activity in Bq and mCi.

$$\lambda = \frac{0.693}{t_{1/2}}$$

$$= \frac{0.693}{1620 \text{ yr}}$$

$$= 4.28 \times 10^{-4} \text{ yr}^{-1}$$

$$= 1.356 \times 10^{-11} \text{ sec}^{-1}$$

$$A = \lambda N$$

$$= 4.28 \times 10^{-4} \text{ yr}^{-1} \times \frac{\text{yr}}{365 \text{ day}} \times \frac{\text{day}}{24 \text{ hr}} \times \frac{\text{hr}}{3600 \text{ sec}} \times 1 \text{ mg} \times 10^{-3} \text{ g} \times \frac{\text{mole}}{226 \text{ g}} \times 6.02 \times 10^{23} \text{ atom}$$

$$= 3.61 \times 10^{7} \text{ atom/sec}$$

$$= 3.61 \times 10^{7} \text{ Bq}$$

$$= 3.61 \times 10^{7} \text{ Bq} \times \frac{\text{Ci}}{3.7 \times 10^{10} \text{ Bq}}$$

$$= 0.976 \times 10^{-3} \text{ Ci}$$

$$= 0.976 \text{ mCi}$$
3. \((J & C 3-4)\) \(^{131}\)I has a half-life of 8.06 d. Find the mean life and the transformation constant. A source of iodine has an activity of 2.5 mCi. Find the activity after 12 days. Express your answer in mCi and Bq.

\[
t_{\text{avg}} = 1.44 \times t_{\frac{1}{2}}
\]
\[
= 1.44 \times 8.06 \text{ day}
\]
\[
= 11.6 \text{ day}
\]
\[
\lambda = \frac{1}{t_{\text{avg}}}
\]
\[
= \frac{1}{11.6 \text{ day}}
\]
\[
= 8.62 \times 10^{-2} \text{ day}^{-1}
\]
\[
= 9.98 \times 10^{-7} \text{ sec}^{-1}
\]

\[
A = A_0 e^{-\lambda t}
\]
\[
= 2.5 \text{ mCi} \times e^{-8.62 \times 10^{-2} \times 12}
\]
\[
= 2.5 \times e^{-0.03} \text{ mCi}
\]
\[
= 0.893 \text{ mCi}
\]
\[
= 0.893 \text{ mCi} \times 10^{-3} \frac{\text{Ci}}{\text{mCi}} \times 3.7 \times 10^{10} \frac{\text{Bq}}{\text{Ci}}
\]
\[
= 3.30 \times 10^7 \text{ Bq}
\]
4. (J & C 3-6) A source of $^{198}$Au with initial activity $10.0 \times 10^3$ Bq is used on a mold and worn by a patient for 5 days. Find the emitted radiation.

The half-life of $^{198}$Au is 2.69 days, so the transformation constant is $0.693/2.69 = 0.258 \text{ day}^{-1}$.

\[
N = \int_0^t \frac{dN}{dt} \, dt \\
= \int_0^t A(t) \, dt \\
= A_0 \int_0^t e^{-\lambda t} \, dt \\
= -\frac{A_0}{\lambda} e^{-\lambda t} \Bigg|_0^T \\
= \frac{A_0}{\lambda} \left(1 - e^{-\lambda T}\right) \\
= \frac{1.0 \times 10^4 \text{ sec}^{-1}}{0.258 \text{ day}^{-1}} \times \frac{24 \text{ hr}}{\text{day}} \times \frac{3600 \text{ sec}}{\text{hr}} \left(1 - e^{-0.258 \times 5}\right) \\
= 3.35 \times 10^9 (1 - 0.275) \\
= 2.43 \times 10^9 \text{ disintegrations}
\]

5. (J & C 3-8) The mass of $^{64}$Cu is 63.9297568 amu. In a $\beta^+$ decay, it is converted into $^{64}$Ni with mass of 63.927956. Find the maximum energy of the $\beta^+$ particle.

\[
\Delta E = (63.9297568 - 63.927956) \text{ amu} \\
= 0.00180 \text{ amu} \times 931 \text{ MeV/amu} \\
= 1.677 \text{ MeV}
\]

Subtracting the 1.022 MeV threshold energy gives us a maximum $\beta^+$ energy of 0.655 MeV.
6. (J & C 3-14) After 100 disintegrations of a source of $^{22}$Na embedded in a small sample of tissue, estimate the energy deposited assuming all gammas escape and that the fluorescent yield is zero. Assume the mean energy of the $\beta^+$ is 1/3 the maximum.

For every 100 disintegrations of $^{22}$Na we produce 90 $\beta^+$ with a maximum energy of 0.54 MeV and 10 characteristic x-rays. If we assume the fluorescent yield is 0, then we can ignore the characteristic x-rays. The energy deposited is then

$$E = \frac{1}{3} \times 90 \times 0.54 \text{ MeV}$$

$$= 16.2 \text{ MeV}$$

7. (J & C 3-18) $^{90}$Rb is a fission fragment with mass 89.91487 amu. Determine the total energy released in the decay of 1 mCi of this isotope into $^{90}$Zr whose mass is 89.9047105 amu.

$^{90}$Rb decays to form $^{90}$Zr with the emission of 3$\beta^-$
(See, for example, the following URL: http://periodictable.com/Isotopes/037.90/index.html)

$^{90}$Rb has a half-life of 2.7 min
(See, for example, the following URL: http://en.wikipedia.org/wiki/Isotopes_of_rubidium),
which is equivalent to a transformation constant of $0.693/2.7 = 0.257 \text{ min}^{-1} = 4.28 \times 10^{-3} \text{ sec}^{-1}$.

So, 1 mCi = $3.7 \times 10^7$ disintegrations sec$^{-1} / 4.28 \times 10^{-3} \text{ sec}^{-1} = 0.864 \times 10^{10}$ disintegrations.

For each disintegration

$$\Delta E = (89.91487 - 89.9047105) \text{ amu}$$

$$= 0.01016 \text{ amu} \times 931 \text{ MeV/amu}$$

$$= 9.459 \text{ MeV}$$

Total energy released = $9.459 \times 0.864 \times 10^{10} = 8.17 \times 10^{10} \text{ MeV}$
8. $^{223}$Ra decays by $\alpha$ emission with 4 $\alpha$s with the following energy spectrum:

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.537</td>
<td>9%</td>
</tr>
<tr>
<td>5.605</td>
<td>26%</td>
</tr>
<tr>
<td>5.714</td>
<td>54%</td>
</tr>
<tr>
<td>5.745</td>
<td>9%</td>
</tr>
</tbody>
</table>

a. What isotope does the $^{223}$Ra decay to?

In $\alpha$ decay, the atomic number is reduced by 2 and the mass number is reduced by 4. Reducing the atomic number by 2 gives us Rn; consequently the decay product is $^{219}$Rn.

b. Sketch the energy level diagram (assuming one $\alpha$ goes to the ground state of the daughter).

![Energy Level Diagram](attachment:energy_level_diagram.png)

223\text{Ra} \quad E=5.745 \text{ MeV} \quad 219\text{Rn} \quad E=5.714 \text{ MeV}

E=5.745 \text{ MeV} \quad E=5.605 \text{ MeV} \quad E=5.537 \text{ MeV}

c. List the possible $\gamma$ rays that could be emitted.

Possible $\gamma$ ray energies are as follows:
- $5.745 - 5.714 = 0.031 \text{ MeV}$
- $5.745 - 5.605 = 0.140 \text{ MeV}$
- $5.745 - 5.537 = 0.208 \text{ MeV}$
- $5.714 - 5.605 = 0.109 \text{ MeV}$
- $5.714 - 5.537 = 0.177 \text{ MeV}$
- $5.605 - 5.537 = 0.068 \text{ MeV}$