Both this lecture and the next lecture are going to address some basic principles of interactions of photons with matter. We are going to defer talking about the specifics of the interactions for later lectures.
In the next two lectures we will try to identify and perform calculations with various kinds of interaction coefficients. We have alluded to interaction coefficients in previous lectures. You may recall we talked about a linear attenuation coefficient when we talked about exponential behavior.

So we are going to reintroduce the linear attenuation coefficient but also talk about a lot of other kinds of interaction coefficients that may have more or less relevance to particular kinds of interactions.

We are going to look at path length and look at various units of path length because path length and interaction coefficients go hand in hand. If we change the interaction coefficient we may have to change the unit of path length so we want to understand what we mean by different kinds of path length.

Then we are going to look at the various interactions that photons undergo with matter and look at their dependences on energy, angle, and atomic number and finally we are going to recognize the implications of various interactions in imaging and in radiation oncology.

Now as I said this is going to be a double lecture. I actually have a lot more information than I want to present in a single lecture.
The basic issue that we are going to look at essentially from here until the end of the course is the idea of “Follow the energy.”

I think all of you heard the phrase “Follow the money,” when people investigated business schemes and the like. Well, in radiation medicine, we want to follow the energy. We want to know where the energy is going. How is the energy distributed in the target material? What can happen to the energy in the target material? How much of it goes here, how much of it goes there? We’re going to be talking about a lot of different quantities that will help us understand better where the energy is going.
Before we do this, however, we need to differentiate between directly and indirectly ionizing radiations. There are some major differences in the kinds of interactions that are undergone when we have directly versus indirectly ionizing radiations.

First of all, directly ionizing radiations are charged particles. Protons, electrons, negative pions, deuterons, are all charged particles. The interactions that these charged particles undergo are Coulomb interactions. An electron in a radiation beam interacts with an orbital electron in target material. It will also interact with a nucleus in the target material. And the interaction is an interaction involving charges. It’s the force that goes as \( \frac{1}{R^2} \).

Consequently, a charged particle will lose a very small amount of its energy as a result of an interaction, and it will then continue on its path to be involved in another interaction. One of the approximations that we make in trying to do calculations involving charged particle interactions is the continuous slowing down approximation (CSDA). What we’re saying is that the amount of energy that’s lost as the result of an interaction is such a small fraction of the initial energy of the particle and these interactions occur at such a rapid rate that we can approximate the energy lost for a charged particle as a continuous function of time.

So we don’t look at discrete interactions when looking at energy loss; we are looking at a continuous loss of energy.

Charged particles transfer energy, in general, at the same rate independent of the energy of the particles. We will see that’s not completely true, but it’s a good approximation. We will also observe some cases where that approximation breaks down. But, to a rough approximation, energy loss is independent of incident energy, and to this same approximation the particle will lose energy continuously until it runs out of energy. Once it runs out of energy it stops.

A consequence of this continuous energy loss is that a charged particle will penetrate target material for only a finite distance. That is the distance until it runs out of energy, and once it runs out of energy it’s not going to penetrate anymore. So it will deposit energy over a finite depth. That’s one of the reasons we are interested in using protons and electrons for radiation therapy. We are only depositing energy for a fixed depth and essentially nothing beyond that depth.
Indirectly ionizing radiations are uncharged particles, and when I say uncharged particles I mean not only neutrons, which are uncharged particles, but also photons. In fact, most of our indirectly ionizing radiation is photons.

When uncharged particles undergo interactions with matter, they first transfer their energy to charged particles. The dominant interaction involving the energy transfer is a catastrophic collision in which a large amount of energy is transferred to the charged particles. So a 1 MeV photon coming in will deposit a large amount of energy, maybe up to 1 MeV, to a charged particle, and it’s now the charged secondary particles that do the remainder of the energy transfer.

The initial collision that an indirectly ionizing particle undergoes is ruled by the laws of probability. That is it is a stochastic process. And we saw that when stochastic processes occur that result in the deposition of energy, these processes are governed by exponential behavior.

So, charged particles deposit energy uniformly up to some depth. Uncharged particles deposit energy in a manner that’s governed by exponential behavior. This is a very important difference between charged particles and uncharged particles.
Now we should note here that directly ionizing radiations could produce either Bremsstrahlung or characteristic x-rays (indirectly ionizing radiations) starting the whole process over again.

So the whole energy deposition process for photons entering a target is quite complicated. We start with an initial catastrophic collision, which produces electrons. It’s also possible that the interaction may generate a scattered photon, which can undergo another catastrophic collision downstream. The electrons deposit energy according to the continuing slowing down approximation, that is uniform deposition of energy, but in the process they may produce Bremsstrahlung or characteristic x-rays, which are indirectly ionizing radiations that can now undergo catastrophic collisions, and so forth.

It’s a very complicated procedure. If you are trying to write computer code that simulates photon and electron interactions in matter, it’s clear that you will have a lot of bookkeeping to do to keep track of what’s going on. This is not an unusual procedure; Monte Carlo calculations are performed to simulate both photon and charged particle interactions with matter. But, photon interactions can be very complicated to keep track of all that is happening.
Let us now talk about attenuation coefficients.

The attenuation coefficient is that quantity that we place in the exponential describing uncharged particle attenuation.

Intensity equals initial intensity multiplied by $e^{-\mu x}$. The quantity $\mu$ is the linear attenuation coefficient.

In general, we can look upon the attenuation coefficient as a measure of the probability of an interaction in a photon beam. A large attenuation coefficient means a high probability of interaction, hence a large amount of attenuation and a small amount of penetration.

In determining the attenuation coefficient, let’s look at the following ideal case. The photon beam is going to be very narrow, the attenuator is thin, the detector is small, and the detector is far away from the attenuator. When we have those four conditions met, we call this narrow beam geometry, or good geometry. Good is a value judgment, there are certain situations where what we call “good geometry” may not be the right geometry. But, certainly for measuring attenuation coefficients narrow-beam geometry is good geometry.
This figure illustrates narrow beam geometry. We have a narrow beam of photons; the attenuator is very thin, so typically we will only have a small number of interaction events. Some of the photons will interact. We want the detector to be small so that we only detect photons that undergo no interaction. If a photon undergoes an interaction, under some circumstances the photon will be completely removed, or absorbed. In other circumstances, it is possible that the photon will be scattered or deflected at a lower energy. We want to make sure when we are doing our measurement that scattered photons are scattered out away from the detector and we are not measuring scattered photons. The only photons that we want to measure are those that undergo no interaction.
Photon interaction

• Let $\Delta N$ be the number of photons interacting
• $\Delta N$ proportional to number of incident photons $N$
  – Proportional to thickness of absorber $\Delta x$
  – Proportional to probability of interaction $\mu$

Let’s try to get some equations together for this.

Let $\Delta N$ be the number of photons interacting. $\Delta N$ is going to be proportional to the number of incident photons. If you double the number of photons, you will double the number of interactions. $\Delta N$ is also going to be proportional to the thickness $\Delta x$ of absorber. Again, remember that we are looking at a thin absorber. If we have a thin absorber, if we double the thickness of absorber, we will also double the number of interactions. Finally the number of photons interacting will be proportional to the probability that an interaction will take place, which is specified by the linear attenuation coefficient $\mu$. 
Finally \( \Delta N \) will be negative, because we are reducing the number of photons in the beam.

Therefore, for a thin absorber, we can write for \( \Delta N \) that \( \Delta N \) is equal to minus \( \mu \) times \( N \) times \( \Delta x \). That’s the equation that governs attenuation in a thin absorber.

We can now write this equation as a differential equation, writing it as \( dN = -\mu N dx \), and integrate it giving us exponential attenuation. Exponential attenuation describes attenuation of photon radiation through a thick target, where we may have multiple interactions. \( N_0 \) is the initial number of photons and it is multiplied by \( e^{-\mu x} \) to give the number of photons passing through absorber thickness \( x \).
Let’s write the attenuation equation in a slightly different form, solving the equation for \( \mu \). The quantity \( \mu \) is equal to minus \( \Delta N \) over \( N \) divided by \( \Delta x \). This way of writing \( \mu \) is really what I would like you to focus on. The reason is that in this manner we can now interpret \( \mu \) as the fraction of photons that interact in an absorber per unit absorber thickness. That is a more physical interpretation of the linear attenuation coefficient: the fraction of photons that interact in absorber per unit absorber thickness.

### Linear attenuation coefficient

- We can write
  \[
  \mu = -\frac{\Delta N / N}{\Delta x}
  \]

- Interpret this as the fraction of photons that interact in absorber per unit absorber thickness
  - This gives us a working definition for linear attenuation coefficient
For example, if $\mu$ is equal to 0.01 per cm, and $\mu$ has the dimensions of reciprocal length, that means that 0.01 or 1% of the beam is attenuated per cm of the absorber.

Now, with 1% being attenuated per cm of absorber, does that mean that in 100 cm, or 1 m, of absorber, 100% of the beam is attenuated? How about if we had 200 cm of absorber, would that mean that 200% of the beam is attenuated? Of course not. If we have 100 cm of absorber, we need to use the thick target approximation instead.
Attenuation is the consequence of a photon interaction. If a photon interacts in any way with target material, either by changing energy or by changing direction, we call it attenuated. It is no longer a part of the primary beam as a result of the attenuation.

We are going to see photons coming out of interactions that have been deflected, that have changed their energy, and that have changed their direction. These are viewed as scattered photons. They have been attenuated because they have undergone an interaction.

The linear attenuation coefficient is the only interaction coefficient we can measure directly; all other attenuation coefficients have to be calculated from the linear attenuation coefficient. We are going to go through a lot of different attenuation coefficients in the course of this lecture and the next lecture. So it’s going to be necessary for you to be really comfortable with these coefficients, to be able to interpret the coefficient as a probability and understand what it is a probability of.
Let’s talk about how to measure attenuation coefficients. In order to do that we measure what is called the half-value layer. The half-value layer is the thickness of attenuator that’s required to reduce the beam intensity to half its original value. It’s the analogy of a half life.

The half-value layer is given by the natural logarithm of 2 divided by the linear attenuation coefficient, or 0.693 divided by the linear attenuation coefficient. Consequently, in order to determine the linear attenuation coefficient, we measure the amount of attenuator required to reduce the beam intensity to half its original value and divide that number into 0.693.

To do this half-value layer measurement, we have to use narrow-beam geometry. In other words, when we do this measurement, we need a thin absorber, a narrow, highly collimated beam, and detector small and far away from the attenuator. This will ensure that scattered photons are deflected away from the detector and only photons that have not interacted will reach the detector.

So, to do this measurement, we set up our measurement configuration, measure the intensity of the unattenuated beam, and start placing attenuator in the path of the beam. We plot beam intensity vs attenuator thickness on a semilogarithmic plot and extrapolate a straight line until the point where the beam intensity is half its original value, and determine the appropriate thickness as the half-value layer.
Another quantity that we sometimes use is called the tenth-value layer. The tenth-value layer is the thickness of attenuator required to reduce the beam intensity to 1/10 its original value. The tenth-value layer is given by the natural logarithm of 10 divided by the linear attenuation coefficient, or $2.303 / \mu = 3.32$ HVL. The tenth-value layer is used a lot in shielding calculations.

One more quantity we use is the mean free path, which is the average distance a photon travels before interacting. The mean free path is $1 / \mu = 1.44$ HVL. The mean free path is analogous to the average life of a radioactive nuclide.
Measurement of attenuation coefficient

- Narrow-beam geometry ensures that (to a good approximation) photons reaching detector have not undergone any sort of interaction

The condition of narrow-beam geometry ensures that photons that reach the detector have not undergone any sort of interaction. This condition is necessary for measurements that characterize the photon beam.
Another configuration, called broad-beam geometry, is sometimes used. Broad-beam geometry is a more realistic case. As we can see on this diagram, when we have broad-beam geometry, some scattered radiation reaches the detector. We have attenuated primary photons reaching the detector as well as scatted radiation reaching the detector.
When we are dealing with broad beam geometry we no longer have purely exponential behavior. The beam intensity $N$ is given by $N_0 e^{-\mu x}$, but this time it is multiplied by a buildup factor $B$, which is a measure of the deviation from purely exponential behavior.
The buildup factor $B$ depends on the photon energy, since scatter is a function of energy. It depends on the attenuator thickness, as a thicker attenuator means we would have more scatter. A thicker attenuator could also mean we have multiple scattering events in which a scattered photon interacts a second time with a target generating another scattered photon.

The buildup factor is a function of the cross-sectional area of the beam; a larger cross-sectional area means more scatter. Finally, the buildup factor is a function of the distance between the attenuator and the detector; the further the detector is from the attenuator, the more likely that scattered radiation will miss the detector. This buildup factor is really a complicated function of a number of factors, and would be very difficult to determine analytically. What is important is a qualitative understanding of how the buildup factor depends on the various parameters.
Does this broad-beam geometry have any practical applications? The answer is yes.

If we are doing shielding calculations, we are trying to calculate the amount of shielding required to protect somebody on the other side of a shielding barrier, then we want to use the worst case scenario. The beam is a therapy beam that’s incident on the wall of the room. We will pick the largest area of the beam; again we want the worst possible case. The wall of the room is thick, so we are going to have more exponential attenuation combined with scatter from the wall. The distance is going to be the distance of somebody’s desk or an operator console from the barrier. We really want to look at the distance from the attenuator being zero, which denotes a situation in which somebody is right up against that wall.

So in the case of shielding calculations, we need to use broad-beam geometry. This buildup factor that modifies exponential behavior is an empirically-determined quantity, which is at least equal to 1 and can go all the way up to a 100. Typically, for a megavoltage therapy room, this buildup factor is about 10-20.

In this situation the amount of radiation that we get from broad-beam geometry is going to be much greater that what we would get from narrow-beam geometry. Consequently if you inadvertently use narrow-beam parameters such as narrow-beam half-value layers for doing a shielding beam calculation saying that’s going to be sufficient, you’re wrong and you are going to under shield your room. So you have to use values that incorporate broad-beam geometry. And those you get out of tables. For example, the National Council on Radiation Protection regularly publishes reports of guidelines for performing shielding calculations for various types of radiation facilities, and provides such tables.

Broad-beam geometry is used for shielding calculations, while narrow-beam geometry is used for beam characterization.
What happens when we are trying to determine the deposition of radiation in a patient? When we are in a patient, our conventional technique for dosimetry ends up separating the beam into two components, a primary component, which has been attenuated, and a scattered component.
The attenuation of the primary beam depends only on the beam energy. The greater the beam energy, the less attenuation of the primary beam. The less the attenuation, the more penetration we will typically have.

When we are looking at scattered radiation we also have to worry about the volume of scatter. How much of the patient is scattering taking place in? So scatter depends on the size of the field, the depth, and also the thickness of the phantom behind the point of interest. When we do these dose calculations we are going to find out that the dose that we calculate is going to be based on beam energy, field size and depth. These are numbers we will get out of tables.

We look at the attenuated primary which only a function of beam energy, and the scatter which is also a function of beam energy but it is also a function of the volume of patient from which radiation is scattered.
The whole procedure for the calculation of photon depth dose quantities in radiation oncology physics is based on this separation of the photon beam into primary plus scatter, and will be dealt with in much more detail in Medical Physics III.
So from this point we are going to look at attenuation coefficients. And we are going to see a lot of different kinds of attenuation coefficients.

Attenuation for photons is any kind of interaction in which the photon changes energy or changes direction. The photon is going to interact with some part of the atom or molecule. This interaction may or may not involve ionization and we are going to see some interactions where there is no ionization. There’s just a change in direction of the photon. The more interesting interactions, however, are where there is ionization.
Let me repeat a statement that I made earlier. Attenuation is a stochastic event. Consequently, the intensity of the beam or the number of photons is going to follow exponential behavior.

We have two quantities in the exponent. The quantity $\mu$ is an attenuation coefficient, which is a measure of the probability of an interaction. The quantity $x$ is a measure of the thickness of the attenuating material. The way in which we express the thickness is not so important, what is important is that we accurately reflect the number of attenuating targets seen by the photons as they pass through the absorber. What is important is that the product $\mu x$ be dimensionless.

First of all, if $\mu$ is expressed as fraction per unit path length, then $x$ must be expressed as a path length. We are going to identify different kinds of attenuation coefficients that express fraction attenuated per unit something. We will then modify the way we express path length so that the product $\mu x$ makes sense. We are going to weight the path length by various physical quantities and we are going to inversely weight the attenuation coefficient by the same quantities.
Here are some examples of attenuation coefficients that we typically use in radiological physics:

The linear attenuation coefficient is a probability of an interaction per unit path length, with dimensions of reciprocal length, reciprocal meters, for example, in SI. If we use a linear attenuation coefficient, then we need to express the path length in units of length, such as meters.

We may use other attenuation coefficients, however. Very often we would rather use a density-weighted attenuation coefficient. This quantity is called the mass attenuation coefficient and is determined by dividing the linear attenuation coefficient by the density of the attenuating material. One of the reasons we will use mass attenuation coefficients is that it will give us probabilities of interaction per unit mass. Ultimately we are going to calculate energy transfer per unit mass. That suggests that we will want to use some coefficient that is related to the mass attenuation coefficient. Mass attenuation coefficient is the linear attenuation coefficient divided by the density with units of area per unit mass, for example, meter squared per kilogram. In order that the argument of the exponential be dimensionless, the path length is weighted by the density, giving us a quantity of mass per unit area. Note the analogy with units of conventional density, which are mass per unit volume. A quantity with units of mass per unit area is referred to as the areal density. We often express attenuation as a mass attenuation coefficient and path length as an areal density.

We are also going to look at some processes that are going to involve interactions of electrons. Photons come in, interact with electrons. Consequently we may want to look at mass attenuation coefficient per electron. This is an electronic attenuation coefficient with units of area per electron. The electronic attenuation coefficient is the mass attenuation coefficient divided by the number of electrons per unit mass. When we do this, we multiply the areal density by the number of electron per unit mass, giving us the number of electrons per unit area, or areal electron density.

How do we determine the number of electrons per unit mass? The number of electrons per unit mass is Avogadro’s number, the number of atoms per mole, multiplied by the atomic number, the number of electrons per atom, divided by the mass number, the mass per mole.

Finally, we will also look at some processes that involve the number of atoms that are present. These processes will be characterized by an atomic attenuation coefficient, which is the mass attenuation coefficient divided by the number of atoms per unit mass. The atomic attenuation coefficient is also the electronic attenuation coefficient weighted by the atomic number. The path length is then the electron areal density divided by the atomic number, or the atomic areal density.

Whatever we do we need the product of the attenuation coefficient and the path length to be dimensionless.

So what I am going to ask you to do between this lecture and the next lecture is to take a good look at this table and try to understand what the implications are. Remember, all of these quantities are probabilities of an interaction. The probability of an interaction per unit something or other. Probability per unit path length, probability per unit density-weighted path length, probability per unit electron, probability per unit atom. So that’s really an important way to look at attenuation coefficients.
Finally, let me remind you of some of the quantities that we use in these definitions: $N_0$ is the number of electrons per unit mass, typically number of electrons per gram. $N_A$ is Avogadro’s number, $6.023 \times 10^{23}$ atoms per mole, $Z$ is the atomic number, or number of electrons per atom, and $A$ is the mass number, or number of grams per mole.