1. (J & C 5.7) Aluminum has a density of 2699 kg/m³. The Compton coefficient per atom is given in Table A-4e for 1.5 MeV photons as $2.232 \times 10^{-28}$ m²/atom. Express this coefficient in m²/electron, cm²/g, m⁻¹. (Use Table 5-3 or the top of Table A-4e.)

To convert from atomic Compton coefficient to electronic Compton coefficient, we must divide the atomic Compton coefficient by the number of electrons per atom, $Z$, which, for aluminum, is 13.

$$\frac{2.232 \times 10^{-28} \text{ m}^2 \text{ atom}^{-1}}{13 \text{ electron atom}^{-1}} = 1.72 \times 10^{-29} \text{ m}^2 \text{ electron}^{-1}$$

To convert to mass Compton coefficient, we must multiply the electronic Compton coefficient by the electron density, which, according to Table A-4e, is $2.902 \times 10^{26}$ electrons kg⁻¹. We must then convert m² to cm² and kg to g with appropriate conversion factors.

$$1.72 \times 10^{-29} \text{ m}^2 \text{ electron}^{-1} \times 2.902 \times 10^{26} \text{ electron kg}^{-1} \times 10^{-3} \text{ kg g}^{-1} \times 10^4 \text{ cm}^2 \text{ m}^{-2}$$

$$= 4.98 \times 10^{-2} \text{ cm}^2 \text{ g}^{-1}$$

To convert to linear Compton coefficient, we must multiply the mass Compton coefficient by the mass density, which is 2699 kg m⁻³. We must convert cm² back to m², and g to kg as well.

$$0.0498 \text{ cm}^2 \text{ g}^{-1} \times 2699 \text{ kg m}^{-1} \times 10^3 \text{ g kg}^{-1} \times 10^{-4} \text{ m}^2 \text{ cm}^{-2}$$

$$= 13.45 \text{ m}^{-1}$$
2. (J & C 5.10) A beam of photons with energy 100 keV suffers Compton collisions. Find the minimum energy of the scattered radiation, the maximum energy the recoil electron may acquire, and the mean energy of the recoil electron.

The maximum energy transfer to the electron will occur when the electron is scattered in a forward direction and the photon is scattered backward. The minimum photon energy is given by

\[ h\nu_{\text{min}} = h\nu \frac{1}{1+2\alpha} \]
\[ = 100 \text{ keV} \frac{1}{1+2\frac{100}{511}} \]
\[ = \frac{100}{1.391} \text{ keV} \]
\[ = 71.9 \text{ keV} \]

The maximum energy of the recoil electron is given by

\[ h\nu_{\text{max}} = h\nu \frac{2\alpha}{1+2\alpha} \]
\[ = 100 \text{ keV} \frac{2\frac{100}{511}}{1+2\frac{100}{511}} \]
\[ = \frac{100 \cdot 0.391}{1.391} \text{ keV} \]
\[ = 28.1 \text{ keV} \]

(We note that if we add the two energies, we get 100 keV, which is the energy of the incident photon beam.)

To obtain the mean energy of the recoil electron, we use J & C equation 6-15.

\[ \sigma \overline{E}_{\nu} = h\nu \frac{\sigma_{\nu}}{\sigma} \]

For 100 keV photons in a free-electron gas, we have

\[ \sigma \overline{E}_{\nu} = 100 \text{ keV} \frac{0.0680}{0.4927} \]
\[ = 13.8 \text{ keV} \]
3. (J & C 5.12) A slab of carbon of thickness $3 \times 10^{23}$ electrons per cm$^2$ is bombarded by $10^6$ photons of energy 1.0 MeV. Calculate the number of Compton interactions, the energy diverted from the beam, the energy transferred to kinetic energy of charged particles, and the energy scattered. Make an energy balance.

The number of photons undergoing no Compton interactions is given by $N = N_0 e^{-\mu \alpha}$, so the number of photons undergoing a Compton interaction is given by $N_C = N_0 (1 - e^{-\mu \alpha})$. From Table A-2a, the Compton coefficient for 1 MeV photons is $0.2112 \times 10^{-28}$ m$^2$ electron$^{-1}$, so

$$\mu \alpha = 0.2112 \times 10^{-28} \frac{m^2}{\text{electron}} \cdot 3 \times 10^{23} \frac{\text{electron}}{\text{cm}^2} \times 10^4 \frac{\text{cm}^2}{\text{m}^2}$$

$$\mu \alpha = 0.0634$$

$$N_C = 10^6 \cdot (1 - e^{-0.0634})$$

$$N_C = 10^6 \cdot (0.0614)$$

$$N_C = 6.14 \times 10^4$$

Note that the target is thick, so it is not completely correct to use a linear approximation.

The energy diverted from the beam is the 1.0 MeV beam energy multiplied by the number of Compton interactions, or $6.14 \times 10^4$ MeV.

The energy transferred to charged particles is the average energy transferred by a Compton interaction multiplied by the number of Compton interactions. The average energy transferred by a Compton interaction for 1.0 MeV photons is given in Table A-2a as 0.440 MeV, so the energy transferred to charged particles is $2.70 \times 10^4$ MeV.

The energy scattered is equal to the energy diverted that is not transferred to charged particles, or $(6.14 - 2.70) \times 10^4$ MeV = $3.44 \times 10^4$ MeV.
4. (J & C 6.10) A detector of area \(2.5 \text{ cm}^2\) is placed 30 cm from a block of scattering material containing \(10^{23}\) electrons/cm\(^2\). A beam of \(10^6\) photons with 1 MeV energy bombards the block. The detector is placed along a line making an angle of 45° with the direction of the photon beam. Find the number of scattered photons that reach the detector (use Fig. 6-4)

The number of scattered photons reaching the detector, \(N\), is the product of the number of incident photons, \(\phi\), the number of electrons in the target, \(N_e\), the differential scatter cross-section, and the solid angle subtended by the detector. The differential scatter cross-section, as taken from Fig 6-4, is \(2.75 \times 10^{-30} \text{ m}^2 \text{ electron}^{-1} \text{ steradian}^{-1}\), for 1 MeV photons scattered an angle of 45°. So,

\[
N = \phi N_e \frac{d\sigma(\theta)}{d\Omega} \Delta \Omega
\]

\[
= 10^6 \cdot 10^{23} \frac{\text{electron}}{\text{cm}^2} \cdot 2.75 \times 10^{-30} \frac{\text{m}^2}{\text{electron} \cdot \text{steradian}} \cdot \frac{2.5}{30^2} \text{steradian} \cdot 10^4 \frac{\text{cm}^2}{\text{m}^2}
\]

\[
= 7.64
\]

5. (J & C 6.11) A beam of \(10^6\) photons with energy 0.8 MeV bombards a block of scattering material containing \(10^{23}\) electrons/cm\(^2\). Find the number of electrons produced with energies in the range of 0.2 to 0.25 MeV (use Fig. 6-5)

The number of electrons produced in an energy interval, \(N\), is the product of the number of incident photons, \(\phi\), the number of electrons in the target, \(N_e\), the differential (energy) scatter cross-section, and the energy interval. The differential (energy) scatter cross section for 0.8 MeV photons in the range of energies 0.2 MeV to 0.25 MeV, as taken from Fig 6-5, is approximately \(38 \times 10^{-30} \text{ m}^2 \text{ electron}^{-1} \text{ MeV}^{-1}\). So,

\[
N = \phi N_e \frac{d\sigma}{dT} \Delta T
\]

\[
= 10^6 \cdot 10^{23} \frac{\text{electron}}{\text{cm}^2} \cdot 38 \times 10^{-30} \frac{\text{m}^2}{\text{electron} \cdot \text{MeV}} \cdot 0.05 \text{ MeV} \cdot 10^4 \frac{\text{cm}^2}{\text{m}^2}
\]

\[
= 1.9 \times 10^3
\]