1. (J & C 5.15) A 10 MeV photon interacts in a pair process. Calculate the energy of the positron if the electron emerges from the interaction with energy of 2.0 MeV.

The energy balance equation is given by

\[ h\nu - 1.022 \text{ MeV} = E_+ + E_-. \]

\[ 10 \text{ MeV} - 1.022 \text{ MeV} = E_+ + 2.0 \text{ MeV} \]

\[ E_+ = 6.978 \text{ MeV} \]

2. (J & C 5.18) The pair process in lead has a cross section of 12.4 \( \times \) 10\(^{-28}\) m\(^2\)/atom at 10 MeV. Find the energy converted into kinetic energy of charged particles when a beam containing \( 10^4 \) photons passes through a block of lead of thickness 1 cm. Assume only pair interactions.

If we assume only pair interactions, then the energy converted to kinetic energy of charged particles is equal to the energy available for conversion (incident energy minus threshold energy) multiplied by the number of photons interacting (exponential attenuation).

Energy available for conversion = 10 MeV – 1.022 MeV = 8.978 MeV

Number of photons interacting = \( N_0(1 - e^{-\mu x}) \).

The linear attenuation coefficient can be found from the cross-section.

\[ \mu = 12.4 \times 10^{-28} \text{ m}^2/\text{atom} \times 2.907 \times 10^{24} \text{ atom/kg} \times 11360 \text{ kg/m}^3 \]

\[ = 40.95 \text{ m}^{-1} \]

\[ \mu x = 40.95 \text{ m}^{-1} \times 10^{-2} \text{ m} \]

\[ = 0.4095 \]

Note that because of the value of \( \mu x \), we need to use the exponential attenuation equation to calculate the number of interactions.

Number of photons interacting = \( 10^4 \times (1 - e^{-0.4095}) = 3360 \) photons

Energy converted = 3360 \times 8.978 \text{ MeV} = 3.02 \times 10^4 \text{ MeV}. 
3. (J & C 5.21) At photon energy of 5 MeV in lead, coherent, Compton, photoelectric, and pair processes all occur. The cross sections in m²/atom are given in columns 2, 3, 4, and 5 of Table A-4i. $10^6$ photons each with energy 5 MeV pass through a foil of lead of thickness $10^{21}$ atom/cm². Find the number of coherent, Compton, photoelectric, and pair processes. Find the mean energy converted to kinetic energy by each process and so determine the mean energy transferred for all processes. Compare with the value in the table.

The number of processes is given by the number of incident photons multiplied by the cross section for the particular process multiplied by the thickness of absorber. Cross-sections for each process in lead are given in J & C Table A-4i.

\[
\begin{align*}
N_{coh} &= 10^6 \cdot 0.0373 \times 10^{-24} \text{ cm}^2 \text{ atom}^{-1} \cdot 10^{21} \text{ atom cm}^{-2} = 0.0373 \times 10^{3} = 37.3 \text{ photons} \\
N_{inc} &= 10^6 \cdot 6.805 \times 10^{-24} \text{ cm}^2 \text{ atom}^{-1} \cdot 10^{21} \text{ atom cm}^{-2} = 6.805 \times 10^{3} = 6805 \text{ photons} \\
N_{photo} &= 10^6 \cdot 0.4096 \times 10^{-24} \text{ cm}^2 \text{ atom}^{-1} \cdot 10^{21} \text{ atom cm}^{-2} = 0.4096 \times 10^{3} = 409.6 \text{ photons} \\
N_{pair} &= 10^6 \cdot 7.288 \times 10^{-24} \text{ cm}^2 \text{ atom}^{-1} \cdot 10^{21} \text{ atom cm}^{-2} = 7.288 \times 10^{3} = 7288 \text{ photons}
\end{align*}
\]

To obtain the kinetic energy transferred, consider the following:

For coherent scatter, there is no energy transfer.

For Compton scatter, Fig 5-8 gives us the mean fraction of energy transfer, which for 5 MeV photons, is approximately 0.62. So the energy transfer is given by $0.62 \cdot 5 \cdot 6805 = 21.096 \times 10^{3}$ MeV

For photoelectric effect, all the energy of the incident photon minus the binding energy of an inner shell electron is transferred to kinetic energy of the photoelectron. For binding energy, we will use the K-shell value of 88 keV. Consequently, the energy transfer is given by $(5 - 0.088) \cdot 409.6 = 2.012 \times 10^{3}$ MeV.

For pair production, all of the energy of the incident photon minus the threshold energy of pair production is transferred to kinetic energy of the positron-electron pair. The energy transfer is given by $(5 - 1.022) \cdot 7288 = 28.992 \times 10^{3}$ MeV

The total energy transfer is given by $21.096 \times 10^{3}$ MeV + $2.012 \times 10^{3}$ MeV + $28.992 \times 10^{3}$ MeV = $52.100 \times 10^{3}$ MeV.

The total number of interactions is given by $0.0373 \times 10^{3} + 6.805 \times 10^{3} + 0.4096 \times 10^{3} + 7.288 \times 10^{3} = 14.540 \times 10^{3}$.

The average energy transfer per interaction is $52.100/14.540$ MeV = 3.58 MeV, which compares with the table value of 3.60 MeV.
4. (J & C 6.19) From Figure 6-7 determine the number of positrons set in motion with energies between 2.0 and 2.5 MeV when a beam of $10^6$ photons of energy 20 MeV impinges on a foil of lead of thickness 0.10 g/cm$^2$.

The total number of pair processes that occur in the lead foil is given by the cross-section for pair production multiplied by the number of incident photons multiplied by the thickness of lead (in atom cm$^{-2}$).

\[ \Delta N = -\kappa N \Delta x \]

\[ = 18.48 \times 10^{-24} \text{ cm}^2 \text{ atom}^{-1} \times 10^6 \text{ photons} \times 0.10 \frac{\text{ g}}{\text{ cm}^2} \times 2.907 \times 10^{24} \frac{\text{ atom}}{\text{ kg}} \times 10^{-3} \frac{\text{ kg}}{\text{ g}} \]

\[ = 5.372 \times 10^3 \text{ photons} \]

The amount of kinetic energy available is 20 MeV – 1.022 MeV = 18.98 MeV.

The fraction of available kinetic energy given to the positrons is $2.5/18.98 = 0.12$, and from Figure 6-7, the relative probability per fractional energy interval for a fraction of available kinetic energy of 0.12 is approximately 1.00. Consequently, the number of positrons set in motion is given by

\[ N = 5.372 \times 10^3 \times 1.00 \left( \frac{2.5 - 2.0}{18.98} \right) \]

\[ = 142 \]
5. (J & C 6.20) A slab of carbon 2 cm thick (density 2.25 g/cm$^3$) is bombarded by $10^6$ photons, each with energy of 20 MeV. Use data from Table A-4b to determine the following:

a. Number of Compton interactions

For 20 MeV photons, the Compton attenuation coefficient is $0.1823 \times 10^{-24}$ cm$^2$ atom$^{-1}$. So, for a thickness of 2 cm,

$$\mu_x = 0.1823 \times 10^{-24} \frac{\text{cm}^2}{\text{atom}} \cdot 2 \text{ cm} \cdot \frac{2.25 \text{ g}}{\text{cm}^3} \cdot \frac{5.014 \times 10^{25} \text{ atom}}{\text{kg}} \cdot \frac{10^{-3} \text{ kg}}{\text{g}}$$

$$= 4.113 \times 10^{-2}$$

Given the value of $\mu_x$, we may use the differential form of the attenuation equation. The number of Compton interactions is thus given by

$$\Delta N = N_0 \mu_x$$

$$= 10^6 \cdot 4.113 \times 10^{-2}$$

$$= 4.113 \times 10^4$$

b. Energy converted to kinetic energy by Compton interactions

The average energy transferred in a Compton interaction at energy 20 MeV is given by Table A-2a to be 14.5 MeV. Consequently the total energy converted to kinetic energy by Compton interactions is given by

$$4.113 \times 10^4 \cdot 14.5 \text{ MeV} = 5.96 \times 10^5 \text{ MeV}.$$ 

c. Energy scattered by the Compton process

The energy scattered by the Compton process is that fraction of the incident photon energy that is not converted to kinetic energy by the Compton interactions, or

$$4.113 \times 10^4 \cdot (20 - 14.5) \text{ MeV} = 4.113 \times 10^4 \cdot 5.5 \text{ MeV} = 2.26 \times 10^5 \text{ MeV}.$$ 

d. Number of pair and triplet processes

For 20 MeV photons, the pair attenuation coefficient is $0.1321 \times 10^{-24}$ cm$^2$ atom$^{-1}$. So, for a thickness of 2 cm,

$$\mu_x = 0.1321 \times 10^{-24} \frac{\text{cm}^2}{\text{atom}} \cdot 2 \text{ cm} \cdot \frac{2.25 \text{ g}}{\text{cm}^3} \cdot \frac{5.014 \times 10^{25} \text{ atom}}{\text{kg}} \cdot \frac{10^{-3} \text{ kg}}{\text{g}}$$

$$= 2.981 \times 10^{-2}$$

The number of pair production interactions is thus given by
\[ \Delta N = N_0 \mu x \]
\[ = 10^6 \cdot 2.981 \times 10^{-2} \]
\[ = 2.981 \times 10^4 \]

**e. Energy radiated as bremsstrahlung**

From Table A-4b, the mean energy transferred is 16.4 MeV, while the mean energy absorbed is 15.3 MeV. Consequently, the mean energy radiated is the difference, or 1.1 MeV.

The total number of interactions is given by \( \Delta N = N_0 \mu x \), where \( \mu \) is the total linear attenuation coefficient.

\[ \Delta N = N_0 \mu x \]
\[ = N_0 \left( \frac{\mu}{\rho} \right) \rho x \]
\[ = 10^6 \text{ photons} \cdot 0.00158 \frac{\text{m}^2}{\text{kg}} \cdot 2250 \frac{\text{kg}}{\text{m}^3} \cdot 2 \text{ cm} \cdot 10^{-2} \frac{\text{m}}{\text{cm}} \]
\[ = 7.11 \times 10^4 \text{ photons} \]

The total energy radiated as bremsstrahlung is then \( 1.1 \text{ MeV} \times 7.11 \times 10^4 \)
\[ = 7.82 \times 10^4 \text{ MeV} \]

**f. Total energy diverted from the beam**

The total energy diverted from the beam is found by multiplying the number of photons involved in interactions by the photon energy of 20 MeV. So, the total energy diverted from the beam is \( 7.11 \times 10^4 \times 20 \text{ MeV} \)
\[ = 142 \times 10^4 \text{ MeV} \]

**g. Total energy converted to kinetic energy**

The total energy converted to kinetic energy is found by multiplying the number of photons involved in interactions by the mean energy transferred, or \( 7.11 \times 10^4 \times 16.4 \text{ MeV} \)
\[ = 116.6 \times 10^4 \text{ MeV} \]

**h. Total energy radiated.**

The total energy radiated is the total energy converted to kinetic energy minus the total energy absorbed. The total energy absorbed is found by multiplying the incident photon energy by the number of photons absorbed. The number of photons absorbed is given by \( \Delta N_{abs} = N_0 \mu_{abs} x \), where \( \mu_{abs} \) is the linear absorption coefficient.
\[
\Delta N_{abs} = N_0 \mu_{abs} x
\]
\[
= N_0 \left( \frac{\mu_{abs}}{\rho} \right) \rho x
\]
\[
= 10^6 \text{ photons} \cdot 0.00121 \frac{m^2}{\text{kg}} \cdot 2250 \frac{\text{kg}}{\text{m}^3} \cdot 2 \text{ cm} \cdot 10^{-2} \frac{\text{m}}{\text{cm}}
\]
\[
= 5.45 \times 10^4 \text{ photons}
\]

Multiplying the number of photons by 20 MeV, gives us the energy absorbed, which is \(5.45 \times 10^4 \times 20 \text{ MeV} = 10.9 \times 10^5 \text{ MeV}\).

Thus, the total energy radiated is the total energy converted to kinetic energy, \(116.6 \times 10^4 \text{ MeV}\), minus the energy absorbed, \(109.0 \times 10^4 \text{ MeV}\), or \(7.6 \times 10^4 \text{ MeV}\).

i. Make an energy balance. Calculate the energy absorbed using \(\mu_{ab}/\rho\) and compare with the energy converted to kinetic energy.

Total energy in beam  = # photons \times \text{energy per photon}
= \(10^6 \times 20.0 \text{ MeV}\)
= \(2.0 \times 10^7 \text{ MeV}\)

Energy transmitted  = (# photons - # interacting) \times \text{energy per photon}
= \((10^6 - 7.11 \times 10^4) \times 20.0 \text{ MeV}\)
= \(9.29 \times 10^5 \times 20.0 \text{ MeV}\)
= \(1.86 \times 10^7 \text{ MeV}\)

Energy absorbed  = \(1.09 \times 10^6 \text{ MeV}\)

Energy re-irradiated  = \(E_{Compt} + E_{PP} + E_{Brems}\)
= \((2.26 \times 10^5 + 2.98 \times 10^4 \cdot 1.022 + 7.82 \times 10^4) \text{ MeV}\)
= \((22.6 + 3.05 + 7.82) \times 10^4 \text{ MeV}\)
= \(33.5 \times 10^4 \text{ MeV}\)

So, Energy transmitted + Energy absorbed + Energy re-irradiated
= \((1.86 + 0.109 + 0.0335) \times 10^7 \text{ MeV}\)
= \(2.00 \times 10^7 \text{ MeV}\)