This is the second of three lectures on Charged Particle Interactions. In the previous lecture, we talked primarily about stopping powers and the various kinds of stopping powers; we derived an equation for stopping powers and now we want to see what some of the consequences are of the stopping powers.
In particular we want to understand what we mean by the range of a charged particle. We want to determine how the range is dependent on energy and understand what happens as a result of a finite range, what happens to the dose delivered by a charged particle beam. Finally, we will look at some of the differences in the dose distribution between electrons and heavy charged particles.
Let us recall the main differences between beams of indirectly ionizing radiation versus beams of directly ionizing radiation as these beams pass through absorbing materials.

We know that for indirectly ionizing radiation, such as photons, the beam loses intensity, that is, the number of particles, because of catastrophic interactions in which a large amount of energy is lost. The depth dependence, then, of the photon beam or of the neutron beam is going to be roughly exponential; the intensity decreases with approximately exponential behavior and will never go to zero.
Uncharged particle interactions are thus differentiated from interactions involving charged particle beams, such as electrons, protons, alphas, etc. When directly ionizing radiations interact with a target, the beam loses energy as a result of interactions in which a small amount of energy is lost. We look upon the loss of energy as a continuous slowing down approximation. That is, we approximate the loss of energy as continuous because the amount of energy loss per interaction is going to be very much less than the energy of the incident particle. In the continuous slowing down approximation, the particle beam finally runs out of energy and stops. Beyond that point there is no more dose except perhaps for contamination. With an electron beam we are going to have Bremsstrahlung contamination because of the radiative interactions, in particular, radiative interactions that occur within the head of the linac. We will have a small amount of x-ray dose coming out of the linac.
We see very major differences in dose distribution differentiating charged particles from indirectly ionizing radiation. For photons, the beam loses intensity, whereas for charged particles, the beam loses energy. For photons, we have exponential attenuation of the intensity of the beam, whereas for charged particles, we have a continuous slowing down approximation governing the loss of energy of the beam. As a consequence, the dose distribution of a charged particle beam does not change significantly with depth, giving us a basically uniform dose. For a photon beam, the intensity, and hence the dose, never goes to zero, whereas for a charged particle beam, there is a finite range beyond which the intensity, and hence the dose, is zero.

This is a very important table for you to keep in mind that differentiates the dose distribution for an electron beam or a proton beam from that of a photon beam.

Again, it’s really important to keep in mind the difference between intensity and energy. A photon beam loses intensity; it does not lose energy. An electron beam loses energy; it does not lose intensity. Be sure to keep these two quantities, intensity and energy, separate.
Because of the fact that a charged particle beam loses some energy as it interacts as it penetrates the patient, the charged particle beam has a finite range. The range of the charged particle beam is the integral of 1 over the energy loss per unit path length until it runs out of energy. We write the range explicitly as the integral of $dE$ from zero energy to $E_0$, the incident energy, of 1 over the stopping power. The units of range are any of the weighted lengths identified previously (depending on units of stopping power).

$$R(E) = \int_0^{E_0} \frac{dE}{S_{tot}(E)}$$
We need to keep in mind that the electron stopping power is nearly constant for all materials from a few hundred keV to at least 20 MeV (where radiative component becomes significant).

- The value of the stopping power is approximately 2 MeV cm$^2$ g$^{-1}$ for all Z, so

$$R(E) \left( \frac{g}{cm^2} \right) = \int_0^{E_0} \frac{dE}{S/\rho} \approx \int_0^{E_0} \frac{dE}{2 [MeV cm^2/g]} = \frac{E_0 [MeV]}{2} \left( \frac{g}{cm^2} \right)$$

Let's calculate the range of the electron beam in g per cm$^2$. The range is the integral from 0 to $E_0$ of dE over S/\rho. We said that S/\rho is a constant equal to 2 MeV cm$^2$ per g. Doing the integration, we find that the range of an electron beam in cm is roughly half the incident energy in MeV. We express the incident energy in MeV and the range is expressed in water-equivalent length, g per cm$^2$. So if we have a 10 MeV electron beam, what is the range of this beam in water?

The answer is 5 cm.
You should now be able to determine the range of an electron beam if I give you the incident energy.

How much lead would it take to shield a 10 MeV electron beam? To determine this we need to know the range of a 10 MeV beam in lead. The range in water is 5 cm, but we need to divide this range by the density of lead. The density of lead is 11.35 g cm\(^{-3}\); for ballpark purposes, we can say it is 10 times the density of water. So, the amount of lead required to shield the 10 MeV electron beam is roughly 0.5 mm. An example of where this might be useful would occur if somebody wants to design an electron treatment of a patient’s head and you need to make a lead shield on the patient’s skin surface that will shield the 10 MeV electrons. In order to do so, your shield must be at least 0.5 mm thick.

That’s another rule of thumb that you will need to keep in mind. The density of lead is roughly 10 times the density of water.
The rule of thumb for electron range is the incident energy over 2 in most materials and over most of the energies that we encounter. That is the range for a few hundred keV to several tens of MeV. The electron energy is expressed in MeV and the density-weighted range in g per cm².

Electron range

- Rule of thumb for electron stopping:

\[ R \left[ \frac{g}{cm^2} \right] = \frac{E_0 [MeV]}{2} \]

- In most materials
- In the energy range from a few hundred keV to several 10s of MeV

- Note: electron energy in MeV and range in density-weighted thickness [g cm⁻²]

The rule of thumb for electron range is the incident energy over 2 in most materials and over most of the energies that we encounter. That is the range for a few hundred keV to several tens of MeV. The electron energy is expressed in MeV and the density-weighted range in g per cm².
It is very important, though, to keep in mind differences in stopping powers between those used to determine range and those used to determine dose. The range of a charged particle beam depends on how the charged particles lose energy. How do they lose energy? Either through collisional processes or radiative processes. Radiative processes are more likely for electrons at higher energies. Range depends on the total stopping power.

Dose, however, arises from ionizations. Dose is the energy deposited locally. So dose is related to the collisional stopping power.

Now that’s not much of a difference when we’re talking about lower energies but as we go to really high-energy electrons we have to keep in mind the difference between range and total stopping power versus dose and collisional stopping power.
Let us calculate the range of an electron beam. To do so, we have to integrate over the inverse of the total stopping power. We can use Johns and Cunningham table 6-3 (water) or table A-5 (various materials) as long as Bremsstrahlung contribution is not too great. So for most energies we are probably OK, but for high-Z materials and higher energies, table A-5 may get us into trouble. You may have to download a table off the internet that includes radiative stopping power.
Let’s do an example of a range calculation. Here’s an example of a table of electron stopping powers extracted from Johns and Cunningham Table 6-3. The first column is electron energy in MeV. The second column is collisional mass stopping powers. The third column is radiative mass stopping powers. Notice that at 10 MeV the radiative stopping power is 10% of the collisional stopping power; at 100 MeV, the two stopping powers are about equal. The fourth column is the total stopping power, which is the sum of collisional and radiative stopping powers. The last column is the range.

<table>
<thead>
<tr>
<th>Electron Energy [MeV]</th>
<th>$S(p)_{\text{col}}$ [MeV cm² g⁻¹]</th>
<th>$S(p)_{\text{rad}}$ [MeV cm² g⁻¹]</th>
<th>$S(p)_{\text{tot}}$ [MeV cm² g⁻¹]</th>
<th>Range [g cm⁻²]</th>
</tr>
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<tbody>
<tr>
<td>0.01</td>
<td>22.56</td>
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<td>0.0003</td>
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<tr>
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<td>4.637</td>
<td>32.47</td>
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</tbody>
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- **Calculate range of 10 MeV electrons in water**
  - From 4 – 10 MeV avg stopping power is $\frac{1}{2}(1.957+2.176) = 2.0665$ [MeV cm² g⁻¹]
  - Partial range is $6$ [MeV]/2.0665 [MeV cm² g⁻¹] = 2.903 cm

Let’s do an example of a range calculation. Here’s an example of a table of electron stopping powers extracted from Johns and Cunningham Table 6-3. The first column is electron energy in MeV. The second column is collisional mass stopping powers. The third column is radiative mass stopping powers. Notice that at 10 MeV the radiative stopping power is 10% of the collisional stopping power; at 100 MeV, the two stopping powers are about equal. The fourth column is the total stopping power, which is the sum of collisional and radiative stopping powers. The last column is the range.

But how would we calculate the range of 10 MeV electrons? Basically we slow the electrons down from 10 MeV down to 0. We look first of all at the interval from 4 to 10 MeV. We start the electrons at 10 MeV, and slow them down to 4 MeV. What is the average stopping power in this energy interval? The average stopping power is the mean of the two total stopping powers. Remember, we are doing a range calculation, so we use total stopping powers. We calculate the average stopping power to be 2.0665 MeV cm² per g. So, what is the distance traveled by the electron as it slows down from 10 MeV to 4 MeV? It’s 6 MeV, the energy interval, divided by the stopping power. So it will travel 2.903 cm as it slows down from 10 MeV down to 4 MeV.
Now, for the next step, we slow the electrons down from 4 MeV to 2 MeV. The average range is 1.9115 MeV cm² per g, and the distance it travels as it slows down from 4 MeV to 2 MeV is 2 MeV divided by 1.9115, or 1.046 cm. We add that to the 2.903 cm to get 3.949 cm.

### Electron range

<table>
<thead>
<tr>
<th>Electron Energy [MeV]</th>
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- **Calculate range of 10 MeV in water**
  - From 2 – 4 MeV avg stopping power is $\frac{1}{2}(1.957+1.866) = 1.9115$ [MeV cm² g⁻¹]
  - Partial range is 2 [MeV]/1.9115 [MeV cm² g⁻¹] = 1.046 cm
  - Add that to 2.903 cm to get 3.949 cm
What do we do then? We slow the electrons down some more from 2 MeV down to 1 MeV. We get the mean value of the stopping power and then divide that into the energy interval of 1 MeV and add that to the distance traveled, and then we integrate from 1 MeV down to 0.4 MeV, 0.4 MeV down to 0.1 MeV, 0.1 down to 0.01 MeV slowing down the electrons so the amount that is the incremental range once we get down to 0.01 MeV is essentially 0. We would find if we did this calculation that we would come pretty close to the 4.917 g per cm² that we see in the table. Doing the range calculation is just basically a straightforward numerical integration.
Let’s talk about the range and depth dose. Recall the shape of the graph of mass stopping power versus energy. At low energy the stopping power is all collisional stopping power. The curve flattens out at around 1 MeV, but at around 10 MeV, we get a little increase in the total stopping power because radiative stopping power starts to become significant.

To determine range we are interested in the total stopping power whereas to determine dose we are interested only in the collisional stopping power.

For heavy charged particles the shape of the stopping power curve is similar, but the contribution from radiative stopping power curve is way down, way below the contribution from collisional stopping power.
The depth dose is the energy deposited as the particle slows down. Keep in mind that the particle is slowing down based on the total stopping power, but the particle deposits dose based on the collisional stopping power.
We’ve looked at plots of stopping power versus energy; now we want to look at plots of relative dose versus path length. We note that the stopping power was plotted on a semi-log scale, but relative dose is typically plotted on a linear scale.

In this graph we see relative dose as a function of path length for a proton beam and an electron beam. Now keep in mind the horizontal axis is path length. We notice for electrons the dose is basically constant until we reach the very end of the path length, and there’s a spike. Why is there a spike? Because of the significant increase in the collisional stopping power. The collisional stopping power really goes up as we get to lower and lower energies. The increase in dose at the end of the range of a charged particle beam is known as the Bragg peak, and notice that for protons we have similar behavior.
But also make note of the fact that the horizontal axis is the path length and not the depth, so we are basically riding an electron tracking how far the electron travels. The electron undergoes a fairly tortuous path. We will look at the dose as a function of depth in a little bit; it is quite different from the dose as a function of path length. But the important thing to keep in mind for both electrons and for protons is dose is very nearly constant until the end of the path when the stopping power increases dramatically and a lot of energy is dumped right at the very end of the path of the charged particles.
There is a definite range beyond which no dose is delivered except for Bremsstrahlung contamination.

For a monoenergetic beam the Bragg peak is very sharp but real beams have energy distributions, and for a real beam the peak is not so high and it’s not so narrow.
The observation of uniform energy deposition in the Bragg peak is not as dramatic for protons as it is for electrons. Notice for protons we have a gradual increase in relative dose as a function of path length, whereas for electrons the dose as a function of path length is basically flat. We have a slightly broader Bragg peak for protons than we do for electrons.

Now it turns out, however, that if we look at the curve of dose versus depth as opposed to dose versus path length, the proton curve for dose versus path length is very similar to that for depth. What that says is that protons are not deflected very much so the path length is roughly the depth.

Keep in mind that we start off with an energy distribution and the loss in energy is a stochastic process. The amount of energy lost in a particular collision cannot be predicted. The energy loss gives rise to a distribution of energies. Consequently, when we start out with a monoenergetic beam, as we penetrate the patient or absorber the energy distribution spreads out a little bit. For electrons, it spreads out more than for protons. This is the phenomenon called straggling, which we discussed earlier in the lecture. Straggling also results from the electrons deviating from a straight line. So this electron dose versus path length curve is really not a good description of the electron dose versus depth. The depth is not really path length at all.
Let’s focus a bit on heavy particle therapy. The basic principle of heavy particle therapy, for example, proton therapy, is that we choose an energy so that the range is appropriate for the depth of the target volume. We determine the maximum depth of the target volume and we pick an energy for the proton beam so that the Bragg peak is placed in the target volume. And generally we’ll spread the energies out by adding some lower energy protons so that the Bragg peak is spread out over the entire target volume. We’ll talk about that when we deal with heavy charged particle therapy.
Here’s a picture of the range of charged particle beams in water. We know what it is for electrons. It’s essentially linear and equal to roughly half the energy. For protons and carbon ions, which are other heavy particles causing some interest in the radiation oncology community, you just are going to have to get familiar with these curves if you plan to do work in this field. The range-energy relationship for these charged particles is, at least to a crude approximation, linear.
Let’s look at the range of a charged particle beam from the equation for $S/\rho$. The mass stopping power is proportional to $Z^2$ times the mass divided by the energy. The range is proportional to $E$ divided by $S/\rho$ so it’s proportional to $E^2$ over $Z^2 \cdot M$. 
Ranges of charged particle beams

- Since $M$ is approximately proportional to $Z$, the range goes approximately as $E^2$ and inversely as $Z^3$.

However, since $M$ is approximately proportional to $Z$, the range of a charged particle beam goes roughly as $E^2$ and inversely as $Z^3$. 
Let’s look at the depth dose curve for heavy charged particles. Heavy charged particles move in a straight line with very little deflection, so all the particles of the same incident energy will stop at the same depth in the patient. The path length and depth are approximately equal for heavy particles. As the particles slow down, the stopping power increases, causing greater ionization density, hence greater dose. This dose enhancement at the end of the range of the heavy particle is the Bragg peak. As a result of the increase in ionization density, there is a greater probability of multiple ionizations in a molecule. As you will learn in your radiation biology course, multiple ionizations are what cause irreparable cell damage. Consequently, at the end of the range of the heavy particle, we see a greater probability of irreparable cell damage, and the relative biological effectiveness of the radiation, the RBE, the efficiency of the radiation to do damage, is greater. When we are looking at protons, we believe there may be an increase in the RBE at the end of the proton range. The conventional RBE for a proton beam is 1.1, meaning it takes 1.1 Gy of photons to cause the same biological damage as 1.0 Gy of protons, but the RBE for protons is likely greater at the end of the range of the protons. We really are not sure of this, so we have a basic rule about proton therapy, that we do not aim the beam so that it stops just before a critical structure.
As a result of the existence of the dose enhancement due to the Bragg peak, and as a result of the possible increase in RBE, there is the possibility of a high therapeutic gain in selecting an energy for the proton beam so that the Bragg peak encompasses the target volume. A Bragg peak from a monoenergetic beam is called a “pristine” Bragg peak. Unfortunately, a pristine Bragg peak is relatively narrow. This would be fine to encompass very small tumors, but most tumors are sufficiently large that a pristine Bragg peak would leave large regions of the tumor underirradiated. For tumors we normally treat in the clinic, we can vary the energy of the proton beam by placing a filter in the head of the proton accelerator. Varying the energy of the proton beam gives rise to a spread-out Bragg peak (SOBP) that can deliver an enhanced dose to the target volume. We normally have a polyenergetic proton beam for treating most tumors. Keep in mind, then, that a pristine Bragg peak comes from a monoenergetic proton beam, whereas a spread-out Bragg peak comes from a polyenergetic proton beam.
We have just looked at the depth dose curve for heavy charged particles. Now, let’s look at the depth dose curve for electrons. Unlike heavy charged particles, electrons do not travel in a straight line. As a result of straggling, the path length is not equal to the depth and the Bragg peak for electrons gets washed out, because it doesn’t occur at the same depth for all electrons. In fact, one of the things we observe because of this electron scatter is a dose buildup below the surface. We get an enhancement of dose a short distance below the surface because the electrons are scattering and as a result, the dose at the surface, where all the electrons are coming in straight, will be less than the dose a bit below the surface where we now have electrons scattering in all directions. This is a very important observation.
As a consequence of this scatter-based buildup, the surface dose is between 75 and 100% of the maximum electron dose. This dose buildup below the surface is due to side-scattered electrons. The average electron is now at an angle with the perpendicular to the surface, so there is a longer path length per unit depth, so more energy is deposited per unit depth, hence a greater dose.

Whereas the dotted line is the idealized electron dose distribution, in reality, the true electron dose distribution is characterized by the solid line.
In a real electron beam, we will typically have a sharp fall-off in dose at the end of the path, but not as extreme a fall-off as with an idealized electron beam. Because of straggling and because of scatter, some of the electrons are going to run out of energy at shallower depths so that the fall-off at end of path is not as sharp as expected from Bragg peak.
Finally, we also see that the electron depth dose distribution exhibits a Bremsstrahlung tail. This tail comes from radiative interactions of electrons primarily occurring in the head of the linear accelerator. In the head, we find higher-Z material, so there is a greater probability of radiative interactions occurring. Also, for higher-energy electrons we have a greater probability of radiative interactions, so the Bremsstrahlung tail tends to be larger for higher-energy beams.
This table summarizes the differences between low-energy electron beams, such as 4 – 6 MeV electrons, and higher-energy electron beams, such as 15 – 20 MeV electrons.

Lower energy beams exhibit a larger amount of side scatter than do higher energy beams. Hence, for a given amount of incident electron fluence, the dose a bit below the surface is going to be greater for low-energy beams. Consequently, the relative percentage of surface dose will be less for low-energy beams than for high-energy beams. As the energy of the beam increases, the percent surface dose increases. For low-energy beams the percent surface dose may be around 75%, whereas for high-energy beams the percent surface dose may be close to 100%. Note that this behavior is different from that of photon beams. For photon beams, increasing the energy decreases the percent surface dose, whereas for electrons, increasing the energy increases the percent surface dose.

<table>
<thead>
<tr>
<th>Low energy</th>
<th>Higher energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large buildup ⇒ large amount of scatter ⇒ lower percent surface dose</td>
<td>Less scatter ⇒ higher percent surface dose</td>
</tr>
<tr>
<td>Smaller range</td>
<td>Greater range</td>
</tr>
<tr>
<td>Rapid fall-off</td>
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</tr>
<tr>
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</tr>
</tbody>
</table>
Low-energy electrons have a shorter range, whereas high-energy electrons have a greater range. The rule of thumb is that the range in water, expressed in cm is half the incident energy in MeV.

Low-energy electrons exhibit a much more rapid fall-off in dose at depths approaching the electron range, whereas for high-energy electrons the fall-off is more gradual.

For low-energy electrons, there is a small amount of Bremsstrahlung contamination, whereas for high-energy electrons, the amount of Bremsstrahlung is greater.

We can explain the energy dependence of the properties of the electron depth dose curve simply by looking at the interactions the electrons undergo with the target material.

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### Features of electron depth dose curves

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We need to know the energy of the electron beam for several reasons. First to know the depth dose characteristics, then to determine the stopping power, which will enable us to convert fluence to dose. One problem with determining the electron energy is that straggling complicates matters considerably, giving us a dispersion of electron energies, so we will need to determine averages over various energy distributions.
This is a very famous graph that originally appeared in ICRU Report #35. It looks at the energy spectrum of an electron beam versus depth in the patient. We start at the point where the electron beam exits the accelerator, at the surface of the patient, and at some depth within the patient. Let’s look at each of these electron spectra more carefully.
At the exit window of the accelerator, we have essentially a monoenergetic electron beam. There is very little spread in the energy spectrum and very little contribution from low-energy electrons. This highly-spiked spectrum is what the electron energies look like immediately after exiting the linear accelerator.
Now, what happens to the beam at the surface of the patient? The beam passes through the accelerator and air, and some interactions occur between the exit window of the accelerator and the surface of the patient. As a result of these interactions, the electron energy is reduced a small amount and the energies are spread out. At the surface of the patient we can talk about a mean electron energy $E_0$. We can also talk about a most probable energy at the surface, which is the maximum point on the spectrum. We can characterize the electron beam by a most probable energy at the surface and by a mean energy at the surface.
Once the beam enters the patient, more interactions take place, and the average energy of the electron is reduced. We represent this energy by $E_z$, the energy at depth $z$, the spectrum is spread out a lot more than at the surface because of the statistical variation in the energy transfer, and we have a large, low-energy component in the spectrum, which will affect the range of the electron beam.
We have seen that the energy of the electron beam is a key parameter that describes its behavior inside the patient. How, then, do we specify the energy of an electron beam?

There are several ways to specify the energy of an electron beam and these ways are all based on a quantity known as the practical range of the electron. The practical range of the electron is a well-defined quantity that can be measured. So, how do we measure the practical range of an electron?

We start with the nominal energy of the electron and we can estimate a range from this nominal energy. Express the energy in MeV and the estimated range of the beam is half the energy in cm. For example, a beam with a nominal energy of 10 MeV will have an approximate range of 5 cm.

But that is not good enough for a precise characterization of the beam.
This is the prescription for measuring the practical range of an electron beam.

First, generate a depth dose curve. This is shown by the dotted line on the graph. Next we have to subtract off the Bremsstrahlung. We can estimate this by extrapolating the Bremsstrahlung tail back toward the surface. That curve is going to be relatively flat. We then draw a line tangent to the depth dose curve at approximately the 50% dose level. The depth corresponding to the point where the two lines intersect is the practical range $R_p$. It's a very simple construction.
Once we have the practical range, we can then specify the energy of the beam.

We’re going to start with the nominal energy of the beam. This is what the linac manufacturer puts on the button that you use to select the beam energy. It’s approximately equal to the energy of the beam at the exit window, for example, 18 MeV. Even though we call it an 18 MeV electron beam, the actual energy may not necessarily be 18 MeV, but it will be close. This is the nominal energy, the energy used to identify the beam.

The practical range, however, is usually identified with the peak energy of the beam at the patient surface. Note that this is not the mean energy at the patient surface, but the peak energy, denoted as \( E_{p,0} \).

---

**Energy specifiers**

- **Nominal energy** – Accelerator manufacturer labels button with approximate energy of beam at exit window, e.g., “18 MeV”
- \( R_p \) is usually identified with peak energy \( E_{p,0} \) of the beam at the patient surface
    \[
    E_{p,0} = 0.48 + 1.95 \ R_p \text{ (MeV)}
    \]
    \[
    E_{p,0} = 0.22 + 1.98 \ R_p + 0.0025 \ R_p^2
    \]
Energy specifiers

- Nominal energy – Accelerator manufacturer labels button with approximate energy of beam at exit window, e.g., “18 MeV”
- $R_p$ is usually identified with peak energy $E_{p,0}$ of the beam at the patient surface
    \[ E_{p,0} = 0.48 + 1.95 R_p \text{ (MeV)} \]
    \[ E_{p,0} = 0.22 + 1.98 R_p + 0.0025 R_p^2 \]

We normally use the Markus relationship to relate $E_{p,0}$ to the practical range. The peak energy in MeV is $0.48 + 1.95$ times the practical range. Note that the peak energy is roughly twice the practical range, which is what it should be. The Markus relationship is a purely empirical relationship.

The Nordic Association of Clinical Physicists use a slightly different formula. They write that the peak energy is $0.22 + 1.98$ times the practical range $+ 0.0025$ times the square of the practical range.

Both are empirical relationships.

A simple exercise would be to compare the two.
It should be pointed out that the more recent protocols are based on the average energy of the electrons at the patient surface, which is related to the depth of 50% dose.

- AAPM Task Group 21 report (5-35 MeV): \( E_0 = 2.33 \ d_{50} \) (\( d_{50} \) is depth of 50% ionization)
- AAPM Task Group 51 report drops all attempts to identify energy, uses depth of 50% dose \( R_{50} \) as beam quality specifier: \( R_{50} = 1.029 \ I_{50} – 0.06 \)

The AAPM Task Group 21 report, which was the standard for dosimetry in the US for many years until around 1990, set the approximate average energy of the electrons at the surface to be equal to 2.33 times the depth of 50% ionization. It took into account the fact that we actually measure ionization rather than dose.

More recently, the AAPM Task Group 51 report, which is the present standard for dosimetry in the US, drops all attempts to identify energy, and uses the depth of 50% dose as the specifier for beam quality. The depth of 50% dose, \( R_{50} \), is related to the depth of 50% ionization, \( I_{50} \), by the relationship \( R_{50} = 1.029 \ I_{50} - 0.06 \).

You are going to see a lot more of the Task Group 51 report in your later courses in medical physics and in your clinical rotations.
If we know the average energy at the surface, it is easy to determine the average energy at depth, assuming the continuous slowing down approximation. The average energy at depth, $E_z$, is equal to the average energy at the surface, $E_0$, times 1 minus the depth divided by the practical range.
Finally, what contribution to dose do we get from Bremsstrahlung? We want to look at the Bremsstrahlung yield. As an electron beam passes through absorbing material, we know there is both collisional energy loss and radiative energy loss. The total energy loss is the sum of the collisional energy loss and the radiative energy loss. The way we determine how much of each is to take the ratio of the appropriate stopping power to the total stopping power. The radiative energy loss is the radiative stopping power divided by the total stopping power times the total energy loss, while the collisional energy loss is the collisional stopping power divided by the total stopping power times the total energy loss.
Bremsstrahlung yield

• Total energy radiated given by

\[
\text{total energy radiated} = \int_0^{E_0} \frac{S_{\text{rad}}(E)}{S_{\text{tot}}(E)} dE
\]

• Fraction energy radiated
  (Bremsstrahlung yield) given by

\[
B = \frac{1}{E_0} \int_0^{E_0} \frac{S_{\text{rad}}(E)}{S_{\text{tot}}(E)} dE
\]

The total energy radiated is the integral over all energies from 0 to \(E_0\), the initial energy, of the radiative stopping power divided by the total stopping power, and the fraction of energy radiated, the Bremsstrahlung yield, is the total energy radiated divided by the initial energy.
Now we can do a calculation. Let us determine the energy radiated and absorbed when a 10 MeV electron is brought to rest in water and in bone.

We need to determine Bremsstrahlung yields, which we can find in table A-6 of Johns & Cunningham. For 10 MeV electrons in water, the Bremsstrahlung yield is 0.0404, or about 4%, while for 10 MeV electrons in bone, the Bremsstrahlung yield is a bit higher, at a little over 5%, or 0.0527. Why is the Bremsstrahlung yield higher in bone? Bone has a higher Z, and we are more likely to have a radiative interaction in material with a higher Z.
In water, the energy radiated is the Bremsstrahlung yield multiplied by the incident energy, or 0.404 MeV, while the energy absorbed is the difference between the incident energy and the energy radiated. 10 MeV minus 0.404 MeV is 9.596 MeV.

In bone, the energy radiated is the Bremsstrahlung yield multiplied by the incident energy, or 0.527 MeV, while the energy absorbed is the difference between the incident energy and the energy radiated. 10 MeV minus 0.527 MeV is 9.473 MeV.

Note, then, that higher energy electrons will deliver a slightly lower dose to bone than to water.

Again, we follow the energy.

Example (J & C 6-12)

- **In water**
  - Energy radiated = 10 MeV x 0.0404 = 0.404 MeV
  - Energy absorbed = 10 MeV – 0.404 MeV = 9.596 MeV

- **In bone**
  - Energy radiated = 10 MeV x 0.0527 = 0.527 MeV
  - Energy absorbed = 10 MeV – 0.527 MeV = 9.473 MeV
The dependence of Bremsstrahlung yield is related to the dependence of radiative vs collisional stopping powers. At higher energy, more radiative energy loss, consequently the Bremsstrahlung yield will be greater. For higher Z materials, more radiative energy loss, consequently the Bremsstrahlung yield will also be greater.