1. A large sheet of lead one half inch thick is placed normal to a neutron beam. What fraction of the neutrons would you expect to get through this plate without absorption?

For Pb, $\sigma_a = 0.17$ barns, $\sigma_s = 11.4$ barns, $\rho = 11.35$ g/cm$^3$, $A = 207.7$ g/mole

The fraction of neutrons that pass through the lead without interaction is given by the exponential attenuation equation:

$$\frac{I}{I_0} = e^{-\Sigma t},$$

where $\Sigma$ is the total macroscopic cross section and $t$ is the absorber thickness. Consequently, the fraction that interacts with the lead is given by

$$\text{frac interacting} = (1 - e^{-\Sigma t}).$$

However, interactions include both absorption and scatter. The fraction of neutrons that are absorbed by the lead is given by

$$\text{frac absorbed} = \frac{\sigma_a}{(\sigma_a + \sigma_s)}(1 - e^{-\Sigma t}),$$

The total macroscopic cross section is given by

$$\Sigma = N(\sigma_a + \sigma_s)$$

$N$ is the atomic density of lead, which is the mass density, 11.35 g/cm$^3$, multiplied by Avogadro’s number, $6.03 \times 10^{23}$ atoms/mole, divided by the atomic weight 207.7 g/mole, or $3.30 \times 10^{22}$ atoms/cm$^3$.

So the total macroscopic cross section is given by

$$\Sigma = 3.30 \times 10^{22}\text{atoms cm}^{-3} \times (0.17 + 11.4) \times 10^{-24}\text{cm}^2$$

$$= 0.38\text{cm}^{-1}$$

The thickness of lead is 0.5 in $\times$ 2.54 cm in$^{-1}$ = 1.27 cm.

The fraction absorbed is then given by

$$\text{frac absorbed} = \frac{0.17}{11.4}(1 - e^{-0.38\times1.27})$$
= 0.0149(1 − e^{-0.483})
= 0.0149(1 − 0.617)
= 0.0057

So, 0.57% of the neutrons are absorbed by the lead, and the percentage of neutrons not absorbed by the lead is 99.43%

2. Dry air has a density of 0.0013 g/cm³ and is roughly 79% nitrogen by weight and 21% oxygen by weight.

For oxygen: \(\sigma_a=0.00027\) barns, \(\sigma_s = 3.76\) barns, \(A=16\) g/mole
For nitrogen: \(\sigma_a=1.85\) barns, \(\sigma_s = 10.6\) barns, \(A=14\) g/mole

a. What is the macroscopic cross section for scattering and for absorption of thermal (2200 m/s) neutrons in air?

The macroscopic cross sections, \(\Sigma_s\) and \(\Sigma_a\), are given by

\[
\Sigma_s(\text{air}) = NO \sigma_s(O) + NN \sigma_s(N)
\]

\[
\Sigma_a(\text{air}) = NO \sigma_a(O) + NN \sigma_a(N)
\]

The atomic density of oxygen is

\[
0.0013 \text{ g/cm}^3 \times 0.21 \times 6.02 \times 10^{23} \text{ atoms/mole} / 16 \text{ g/mole}
\]

\[= 1.03 \times 10^{19} \text{ atoms/cm}^3\]

while the atomic density of nitrogen is

\[
0.0013 \text{ g/cm}^3 \times 0.79 \times 6.02 \times 10^{23} \text{ atoms/mole} / 14 \text{ g/mole}
\]

\[= 4.42 \times 10^{19} \text{ atoms/cm}^3\]

So the macroscopic cross sections are

\[
\Sigma_s(\text{air}) = 1.03 \times 10^{19} \text{ atoms/cm}^3 \times 3.76 \times 10^{-24} \text{ cm}^2 + 4.42 \times 10^{19} \text{ atoms/cm}^3 \times 10.6 \times 10^{-24} \text{ cm}^2
\]

\[= 5.07 \times 10^{-4} \text{ cm}^{-1}\]

\[
\Sigma_a(\text{air}) = 1.03 \times 10^{19} \text{ atoms/cm}^3 \times 0.00027 \times 10^{-24} \text{ cm}^2 + 4.42 \times 10^{19} \text{ atoms/cm}^3 \times 1.85 \times 10^{-24} \text{ cm}^2
\]

\[= 8.18 \times 10^{-5} \text{ cm}^{-1}\]
b. What percentage of the neutrons absorbed in air is absorbed in oxygen?

The percentage of neutrons absorbed in oxygen is given by

\[
pct = \frac{\Sigma_a(O)}{\Sigma_a(air)}
\]

\[
= \frac{1.03 \times 10^{19} \times 0.00027 \times 10^{-24}}{8.18 \times 10^{-5}}
\]

\[
= 0.0034\%
\]

c. What is the probability of a neutron traveling 100 m in air without reacting?

The probability of no interaction is given by

\[
prob = e^{-(\Sigma_a + \Sigma_s)t}
\]

\[
= e^{-\left[(8.18 \times 10^{-5}\text{cm}^{-1} + 5.07 \times 10^{-4}\text{cm}^{-1}) \times 100 \text{m} \times 100 \text{cm/m}\right]}
\]

\[
= 0.0028
\]

3. When a 0.0025 cm thick foil of material of mass number 10 and density 2 g/cm³ is bombarded with a neutron beam perpendicular to foil plane, it is observed that only 50% of the neutrons pass through. Assuming that scattering is negligible, calculate the microscopic cross section for neutron absorption of this material.

The fraction of neutrons passing through an absorber is given by the equation

\[
\frac{I}{I_0} = e^{-\sigma N t}
\]

Rearranging, we have

\[
\log \frac{I}{I_0} = -\sigma N t
\]

In this equation log means natural logarithm.

Rearranging, we get

\[
\sigma = \frac{\log \left(\frac{I_0}{I} \right)}{N t}
\]

Note that since we assumed that scattering is negligible, \( \sigma_{tot} = \sigma_a \).
\[ N = \frac{2 \text{ g/cm}^3 \times 6.02 \times 10^{23} \text{ nuclei/mole}}{10 \text{ g/mole}} = 1.20 \times 10^{23} \text{ nuclei/cm}^3 \]

So,

\[
\sigma_a = \frac{\log 2}{1.20 \times 10^{23} \text{ nuclei/cm}^3 \times 0.0025 \text{ cm} \times 1 \text{ barn/}10^{-24} \text{ cm}^3} = 2310 \text{ barn}
\]

4. A certain medium has the following properties: \( \Sigma_s = 0.1 \text{ cm}^{-1} \) and \( \Sigma_a = 0.01 \text{ cm}^{-1} \).

a. What is the probability that a neutron will travel 1 cm in this medium without interacting with a nucleus?

The probability that a neutron will travel a specified distance without an interaction is given by

\[
\frac{I}{I_0} = e^{-(\Sigma_a + \Sigma_s)t} = e^{-0.11 \times 0.1}
\]

\[ = 0.896 \text{ or } 89.6\% \]

b. If the neutron speed is \( 2 \times 10^5 \text{ cm/s} \), what is the average time between scattering collisions?

The time between collisions is the distance the neutron travels between collisions (the mean free path) divided by the speed of the neutron. The mean free path is the reciprocal of the macroscopic scatter cross section:

\[ \lambda_s = \frac{1}{\Sigma_s} \]

\[ = \frac{1}{0.1 \text{ cm}^{-1}} = 10 \text{ cm} \]

The average time between collisions is then given by

\[ \bar{t} = \frac{\lambda_s}{\nu} = \frac{10 \text{ cm}}{2 \times 10^5 \text{ cm/s}} \]

\[ = 5 \times 10^{-5} \text{ s} \]