4.1a - Cavity Theory Lecture 1

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More rigorous than Johns and Cunningham
Treatment of cavity theory used in this course
Clinical Dosimetry Measurements in Radiotherapy. D.W.O. Rogers, Joanna E Cygler, Editors
Chapter 3 by Alan Nahum
‘Cavity Theory, Stopping Power ratios, Correction Factors’
Most up-to-date and clear summary of cavity theory available

Perhaps one of the greatest contributions physics has made to radiation oncology and radiology has been in developing ways to measure radiation accurately and precisely.
Tumor Control Probability, TCP, curves
Normal Tissue Complication Probability, NTCP, curves
Cell survival curves
Clinical results presented versus the dose given.
Reducing dose in diagnostic techniques.
Etc.
Only has meaning if the dose is accurately and precisely measured and can be related to a national standard.

Accuracy is a statement of how close the value measured is to the true value of the quantity being measured.

Precision is a statement of the reproducibility of the measurements and has to do with random errors due to fluctuations in the instrument characteristics, ambient conditions, etc.
Precision can be determined by repeat measurements and finding the standard deviation.
High precision is associated with a small standard deviation.
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When making a measurement of a quantity there is usually a national or international standard against which the measurement can be compared, e.g. mass, the kilogram; length, meter; time, sec; etc. For radiation there is no standard beam against which to compare other radiation beams or against which we can calibrate instruments. When this problem was realized a measure of radiation was introduced, for x-rays and γ-rays only, the unit for which was the roentgen, defined as the amount of radiation that produces 1 e.s.u. of charge in 1 cm$^3$ of air at S.T.P. The quantity for which the roentgen is the unit was later defined as exposure.

This meant that the standard for the unit is defined by the measurement so the standard became the measuring instrument. The Standard Free Air Ionization Chamber Beams were defined by their H.V.L. but the standard was the chamber. This worked as long as the x-ray energies were not too high but between 300kev and 400kev x-rays it became impossible to build a free air ionization chamber that could determine the roentgen to the desired accuracy.

When cobalt-60 and 2 MV x-rays came along a different way of determining exposure had to be found. One of the first applications of cavity theory was to determine exposure in terms of roentgens for these energies. But x-ray energies went higher, up to 20MV-30 MV, with first betatrons, then linear accelerators, Electrons at energies in the 4 Mev to 25 Mev Neutrons Protons The quantity exposure was not applicable to these energies and modalities.
Since exposure was not applicable, a new quantity was introduced: absorbed dose.

The unit for absorbed dose was the rad defined as:
100 erg/gm absorbed in a medium.

This is not an SI unit so later it was changed to the gray
1 Gy = 1 J/kg
and
100 rad = 1 Gy

What dosimeters can be used to measure absorbed dose? In particular to measure absorbed dose absolutely.

An absolute dosimeter is one that can measure absorbed dose without requiring calibration in a known radiation field.

Three types of dosimeters are regarded as being capable of absoluteness. They are:
Calorimeters
Cavity Ionization Chambers
Fricke Ferrous sulfate dosimeters (chemical dosimeter)

Calorimeters measure heat directly-temperature- therefore the energy deposited.
Ionization chambers and chemical dosimeters depend upon coefficients of conversion: ionization (W), chemical yield (G).
The general problem:
A detector is placed in a medium.
Irradiated with either photons, electrons or protons to a given quantity of radiation "exposure", corresponding to a known number of monitor units (MU) from a linac, or a known length of time, $t$ (sec) from a cobalt unit.
What is the absorbed dose $D_{med}$ in the medium in the absence of the detector?
Relative measurements i.e. ratio of dose at one point in medium to dose at another point in the medium.
Absolute Dose, absorbed dose in gray (Gy)

Requirement for the detector: its signal must be proportional to the energy absorbed in the detector and therefore the absorbed dose in the detector, $D_{det}$, which in general will be a different material than the medium. Therefore if the mass of the detector changes it must be taken into account. (e.g. temperature pressure correction).
First step is to relate the raw detector signal $M_{det}$ to $D_{det}$ at a reference radiation quality is called calibration.
For an air-filled ion chamber
$D_{air} = \left(\frac{Q_{air}}{m_{air}}\right) \times \left(\frac{W_{air}}{E}\right)$
The detector can be thought of as a cavity in the medium, the concept derived when gas-filled ionization chambers were the detectors. The associated theory which relates $D_{\text{det}}$ to $D_{\text{med}}$ is known as cavity theory.

In its most general form

$$f(Q) = \left[ \frac{D_{\text{med}}}{D_{\text{det}}} \right] Q$$

For an arbitrary "det" in an arbitrary medium "med", for an arbitrary radiation quality $Q$.

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For photons

Under charged particle equilibrium (CPE) the absorbed dose in the medium $D_{\text{med}}$, is related to the photon energy fluence in the medium $\Psi_{\text{med}}$.

$$D_{\text{med}}(\text{CPE}) = \Psi_{\text{med}}(\mu_{\text{en}}/\rho_{\text{med}})$$

For charged particles (electrons)

Under delta-ray equilibrium, $\delta$-eqm. the absorbed dose in the medium $D_{\text{med}}$, is related to the particle fluence in the medium $\Phi_{\text{med}}$.

$$D_{\text{med}}(\delta\text{-eqm.}) = \Phi_{\text{med}}(S_{\text{col}}/\rho_{\text{med}})$$
For a photon detector at energy $Q$ :

$$D_{\text{med}1} = D_{\text{med}2} \cdot \frac{\mu_{\text{en}}}{\rho_{\text{med}1}}$$

$$f(Q) = \frac{\mu_{\text{en}}}{\rho_{\text{med}1}} = \frac{\mu_{\text{en}}}{\rho_{\text{med}2}}$$

The averaging is over the photon spectrum in the media, and the spectrum needs to be the same in both media. med1 is the medium of interest, med 2 is the detector medium.

For charged particles (electrons) $E$ :

$$D_{\text{med}1} = D_{\text{med}2} \cdot \frac{S_{\text{ion}}}{\rho_{\text{med}1}}$$

$$f(E) = \frac{S_{\text{ion}}}{\rho_{\text{med}1}} = \frac{S_{\text{ion}}}{\rho_{\text{med}2}}$$

The averaging is over the electron spectrum in the media, and the spectrum needs to be the same in both media. med 1 is the medium of interest, med 2 is the detector medium.

**Overview of Lecture**

- Definitions
- Energy Transferred
- Net Energy Transferred
- Collision Kerma
- Energy Imparted
- Absorbed Dose
• Ionization
  Ionization is a process in which one or more electrons are liberated from a parent atom or molecule or other bound state.

• Ionizing Radiation
  Ionizing Radiation consists of charged particles (for example, positive or negative electrons, protons, or other heavy ions) and/or uncharged particles (for example, photons or neutrons) capable of causing ionization by primary or secondary processes.

• Directly and Indirectly Ionizing Radiation
  1. Directly Ionizing Radiation
     Fast charged particles, which deliver their energy to matter directly, through many small Coulomb-force interactions along the particle’s track.

  2. Indirectly Ionizing Radiation
     X- or γ-ray photons or neutrons (i.e., uncharged particles), which first transfer their energy to charged particles in the matter through which they pass in a relatively few large interactions. The resulting fast charged particles then in turn deliver the energy to the matter as above.
     It will be seen that the deposition of energy in matter by indirectly ionizing radiation is thus a two-step process.
Stochastic and Nonstochastic Quantities

Stochastic quantities have to do with random values and any particular value is determined by a probability distribution. Radiation interactions are random in nature so quantities that we deal with are often stochastic quantities. The expectation value of a stochastic quantity is the mean of its measured values as the number of observations approaches \( \infty \).

Stochastic and Nonstochastic Quantities

Continued

Stochastic quantities vary discontinuously in space and time, meaningless to speak of gradient and rate of change. For dosimetry we like nonstochastic quantities so spatial gradients and time rate of changes can be considered. In the context of ionizing radiation nonstochastic quantities are related to stochastic quantities by their expectation values.

Stochastic and Nonstochastic Quantities

Continued

Radiation dosimetry is nonstochastic dosimetry. Stochastic dosimetry is called micro-dosimetry.
Particle Fluence

- The ICRU defines particle fluence, \( \Phi \), as the quotient of \( dN \) by \( da \), where \( dN \) is the number of particles incident on a sphere of cross-sectional area \( da \).
- Units are \( m^{-2} \).

\[ \Phi = \frac{dN}{da} \]

Particle Fluence

- Care must be taken to distinguish fluence from planar fluence, which is the number of particles crossing a plane per unit area.
- In the two cases shown, the particle fluence is the same because the number of particles hitting the spheres is the same in both cases, whereas the planar fluence decreases when the beam is not at normal incidence.

Radiant Energy

ICRU defines radiant energy as the energy of particles (excluding rest energy) emitted, transferred, or received.

\( R = NE \)

Units are J.
Energy Fluence

- The energy fluence, $\Psi$, is the quotient of $dR$ by $da$, where $dR$ is the radiant energy incident on a sphere of cross-sectional area $da$.
- Units are Jm$^{-2}$. $\Psi = \frac{dR}{da}$

The change of fluence $\Phi$ with time is called fluence rate or flux density $\varphi$

$$\varphi = \frac{d\Phi}{dt}$$

The change of energy fluence $\Psi$ with time is called energy fluence rate or energy flux density $\psi$

$$\psi = \frac{d\Psi}{dt}$$

Energy Transferred

$$\varepsilon_s = (R_{\text{th}}) - (R_{\text{alb}}) + \Sigma \alpha$$

$$-h\nu_1 - h\nu_2 + 0 = T$$
Energy Transferred, \( \epsilon_{tr} \)

- The energy transferred in a volume \( V \) is, by definition
  \[
  \epsilon_{tr} = (R_{in})\nu - (R_{out})\nu_{nor} + \Sigma Q,
  \]
  where \( (R_{in})\nu \) is the radiant energy of uncharged particles entering \( V \) (and radiant energy is the particle's energy ignoring the rest mass), \((R_{out})\nu_{nor}\) is the radiant energy of uncharged particles leaving \( V \), except that which originated from radiative losses of kinetic energy by charged particles while in \( V \) (i.e., except for the bremsstrahlung originating in \( V \)), and \( \Sigma Q \) is the net energy derived from rest mass in \( V \).

Energy Transferred, \( \epsilon_{tr} \)

- \( \epsilon_{tr} \) is just the kinetic energy received by charged particles in the volume \( V \), regardless of how they dissipate the energy.
  \[
  \epsilon_{tr} = (R_{in})\nu - (R_{out})\nu_{nor} + \Sigma Q
  \]
  \[
  = h\nu_1 - h\nu_2 + \phi = \gamma
  \]

Kerma

- The kerma \( K \) at a point of interest \( P \) in a volume \( V \) is defined as
  \[
  K = d\epsilon_{tr}/dm
  \]
  The kerma is the expectation value of the energy transferred to charged particles per unit mass at a point of interest.
  Kerma is a nonstochastic quantity.
  Unit:J/kg. 1JK/kg = 1Gy

Kerma is defined for indirectly ionizing radiation only (i.e., photons and neutrons).
Mass Energy Transfer Coefficient

- The mass energy transfer coefficient, $\mu_t/\rho$, of a material for uncharged ionizing particles, is the quotient of $dE_{\text{r}}/EN$ by $\rho dl$, where $E$ is the energy of each particle (excluding rest energy), $N$ is the number of particles, and $dE_{\text{r}}/EN$ is the fraction of incident particle energy that is transferred to kinetic energy of charged particles by interactions in traversing a distance $dl$ in the material of density $\rho$.
- Units are $m^2 kg^{-1}$.

\[
\mu_t = \frac{1}{\rho} \frac{dE_{\text{r}}}{EN dl}
\]

Kerma

- For monoenergetic photons, the kerma is related to the energy fluence $\Phi$ by
  \[
  K = \Phi \left(\frac{\mu_t}{\rho}\right)
  \]
  where $(\mu_t/\rho)$ is the mass energy transfer coefficient.

Net Energy Transferred, $\varepsilon_{tr'}^n$

\[
\varepsilon_{tr'}^n = (R_{in})_r - (R_{out})_{nonr} - R_{in} - \Delta \Omega
\]
\[
= h_{W} - h_{S} - (h_{W} + h_{S})
\]
\[
= T - (h_{W} + h_{S})
\]
Collision Kerma

- Let the net energy transferred be defined for a volume $V$ as
  \[ e_n = (R_{in})_U - (R_{out})_{new} - R'_{in} + \Delta Q \]
  where $R'_{in}$ is the radiant energy emitted as radiative losses by the charged particles that originated in $V$, regardless of where the radiative loss events occur. Thus $e_n$ does not include energy going into radiative losses, whereas $e_r$ does.

Collision Kerma

- The collision kerma is defined as
  \[ K_c = \frac{d e_n}{d m} \]

  The collision kerma is the expectation value of the net energy transferred to charged particles per unit mass at the point of interest, excluding the radiative loss energy.

Mass Energy Absorption Coefficient

- The mass energy absorption coefficient, $\mu_{en}/\rho$, of a material for uncharged ionizing particles, is the product of the mass energy transfer coefficient, $\mu_r/\rho$, and $(1-g)$, where $g$ is the fraction of the energy of secondary charged particles that is lost to bremsstrahlung in the material.
- Units are $m^2/kg^{-1}$.
  \[ \frac{\mu_{en}}{\rho} = \frac{\mu_r}{\rho} (1 - g) \]
Collision Kerma

- For monoenergetic photons, the collision kerma is related to the energy fluence $\Psi$ by
  \[ K_c = \Psi(\mu_{ne}/\rho) \]
  where $(\mu_{ne}/\rho)$ is the mass energy absorption coefficient.

Absorbed Dose

- Let the energy imparted by ionizing radiation to matter of mass $m$ in a finite volume $V$ be defined as
  \[ \varepsilon = (R_{ou}) + (R_{in}) + (R_{ou}) + \sum Q \]
  where $(R_{ou})$ is the radiant energy of all uncharged radiation leaving $V$, and $(R_{in})$ and $(R_{ou})$ are the radiant energies of the charged particles entering and leaving $V$.

Absorbed Dose

- The absorbed dose $D$ at any point $P$ in $V$ is defined as
  \[ D = d\varepsilon/dm \]
  Thus, the absorbed dose $D$ is the expectation value of the energy imparted to matter per unit mass at a point.
  Absorbed dose is a nonstochastic quantity
  Unit: J/kg 1J/kg = 1Gy
Energy Imparted

\[ \varepsilon = (R_{in})_c - (R_{out})_c + (R_{in})_c - (R_{out})_c + 2\Omega \]
\[ = h_{W_1} + h_{W_2} + d - T' + \theta \]

\((R_{in})_c = (R_{out})_c\)

Is called charged

Particle equilibrium

CPE

With CPE

\[ \varepsilon' = \varepsilon \]
\[ D = \varepsilon \]
\[ D = \Psi(\mu/n) \]