1. An electron enters a volume $V$ with kinetic energy $4 \text{ MeV}$, and carries $0.5 \text{ MeV}$ of that energy out of $V$ when it leaves. While in the volume it produces a Bremsstrahlung x-ray of $1.5 \text{ MeV}$, which escapes from $V$. What is the contribution of this event to:

(a) The energy transferred?

The energy transferred is the energy transferred from an incident photon to kinetic energy of electrons. Because the energy entering the volume $V$ comes from a charged particle, rather than from a photon, the energy transferred is zero.

(b) The net energy transferred?

The net energy transferred is the energy transferred from an incident photon to kinetic energy of electrons minus the energy lost to radiative processes. Because the energy entering the volume $V$ comes from a charged particle, rather than from a photon, the net energy transferred is zero.

(c) The energy imparted?

The energy imparted is given by

$$\epsilon = (R_{in})_u - (R_{out})_u + (R_{in})_c - (R_{out})_c + \Sigma Q$$

$(R_{in})_u$ is the radiant energy of uncharged particles entering the volume and $(R_{in})_c$ is the radiant energy of charged particles entering the volume. We only have a charged particle of energy $4 \text{ MeV}$ entering the volume, so $(R_{in})_u = 0$, and $(R_{in})_c = 4 \text{ MeV}$. $(R_{out})_u$ is the radiant energy of uncharged particles exiting the volume and $(R_{out})_c$ is the radiant energy of charged particles exiting the volume. We have a photon of energy $1.5 \text{ MeV}$ exiting the volume and a charged particle of energy $0.5 \text{ MeV}$ exiting the volume, so the total energy exiting the volume is $2.0 \text{ MeV}$.

There is no net energy deriving from rest mass.

Consequently, the energy imparted is $4.0 - 2.0 \text{ MeV}$, or $2.0 \text{ MeV}$.

2. (Attix 4-3) A $10-\text{MeV}$ photon enters a volume $V$ and undergoes pair production, thereby disappearing and giving rise to an electron and positron of equal energies.
The electron spends half its kinetic energy in collision interactions before escaping from V. The positron spends half of its kinetic energy in collisions in V before being annihilated in flight. The resulting photons escape from V. What is the contribution of this event to:

(a) The energy transferred?

The energy transferred is given by

\[ \epsilon_{tr} = (R_{in})_u - (R_{out})_{nonr} + \Sigma Q \]

\( (R_{in})_u \) is the radiant energy of uncharged particles entering the volume, \( (R_{out})_{nonr} \) is the radiant energy of uncharged particles leaving the volume, except those originating from radiative losses, and \( \Sigma Q \) is the net energy derived from rest mass in V. The radiant energy of uncharged particles entering the volume is 10 MeV. The radiant energy of uncharged particles leaving the volume not resulting from radiative losses is 1.02 MeV. The net energy derived from rest mass in V is 0, since both the pair is created and a positron is annihilated.

Consequently the energy transferred is 10 – 1.02 MeV, or 8.98 MeV.

(b) The net energy transferred?

The net energy transferred is given by

\[ \epsilon_{tr}^n = (R_{in})_u - (R_{out})_{nonr} - R_{u}^r + \Sigma Q \]

\( (R_{in})_u \) is the radiant energy of uncharged particles entering the volume, \( (R_{out})_{nonr} \) is the radiant energy of uncharged particles leaving the volume, except those originating from radiative losses, \( R_{u}^r \) is the radiant energy of uncharged particles leaving the volume due to radiative losses, and \( \Sigma Q \) is the net energy derived from rest mass in V. The radiant energy of uncharged particles entering the volume is 10 MeV. The radiant energy of uncharged particles leaving the volume not resulting from radiative losses is 1.02 MeV. Since the positron was annihilated in flight with half its initial kinetic energy, the radiant energy of uncharged particles leaving the volume and resulting from radiative losses is equal to the remaining half of the initial kinetic energy of the positron. This value is \( \frac{1}{4} \) of the total energy given to the electron-positron pair, \( 0.25 \times 8.98 \text{ MeV} = 2.25 \text{ MeV} \). The net energy derived from rest mass in V is 0, since both the pair is created and a positron is annihilated.

Consequently the energy transferred is 10 – 1.02 MeV – 2.25 MeV, or 6.73 MeV.

(c) The energy imparted?
The energy imparted is given by

$$\varepsilon = (R_{in})_u - (R_{out})_u + (R_{in})_c - (R_{out})_c + \Sigma Q$$

$(R_{in})_u$ is the radiant energy of uncharged particles entering the volume and $(R_{in})_c$ is the radiant energy of charged particles entering the volume. We only have an uncharged particle of energy 10 MeV entering the volume, so $(R_{in})_u = 10$ MeV, and $(R_{in})_c = 0$ MeV. $(R_{out})_u$ is the radiant energy of uncharged particles exiting the volume and $(R_{out})_c$ is the radiant energy of charged particles exiting the volume. We have two annihilation photons with total energy $1.022$ MeV + $2.25$ MeV exiting the volume and a charged particle of energy $2.25$ MeV exiting the volume. There is no net energy deriving from rest mass.

Consequently, the energy imparted is $10.0 - 1.02$ MeV – $2.25$ MeV – $2.25$ MeV, or 4.48 MeV.

3. Kerma can be divided into two parts:

$$K = K_{coll} + K_{rad}$$

The expression for $K_{coll}$ in terms of energy fluence and the mass energy absorption coefficient was given in class.

Derive the expression for $K_{rad}$ in terms of energy fluence, mass energy absorption coefficient, and $g$, the fraction of the energy of the secondary charged particles that is lost to Bremsstrahlung.

$$K_{rad} = \Psi g(\mu_{tr}/\rho)$$

$$K_{coll} = \Psi (1 - g)(\mu_{tr}/\rho)$$

$$= \Psi (\mu_{en}/\rho)$$

Solving for $(\mu_{tr}/\rho)$, we have

$$(\mu_{tr}/\rho) = (\mu_{en}/\rho)[1/(1 - g)]$$

So,

$$K_{rad} = \Psi (\mu_{en}/\rho)[g/(1 - g)]$$

4. (Attix 4-6) Consider a beam of 3 MeV gamma-rays perpendicularly incident on a Fe foil that is very thin in comparison with the range of the secondary electrons.

(a) What are the values of $K$, $K_{coll}$, and $K_{rad}$ in the foil $(Z=26)$ for a fluence of $5.6 \times 10^{15}$ photons/m$^2$ given that for calcium $(Z=20)$ and Cu $(Z=29)$.
\( (\mu_{tr}/\rho) \) is 0.00221 and 0.00211 m\(^2\)/kg respectively and \( (\mu_{en}/\rho) \) is 0.00216 and 0.00204 m\(^2\)/kg respectively?

For a fluence of \( 5.6 \times 10^{15} \) photons/m\(^2\) of 3 MeV photons, the energy fluence \( \Psi \) is given by

\[
\Psi = 5.6 \times 10^{15} \text{ photons/m}^2 \times 3 \text{ MeV} \times 1.602 \times 10^{-13} \text{ J/MeV}
\]

\[
= 2.69 \times 10^3 \text{ J/m}^2
\]

Approximating the attenuation coefficients to be linearly dependent on \( Z \), we estimate that for Fe, \( (\mu_{tr}/\rho) = 0.00215 \) m\(^2\)/kg and \( (\mu_{en}/\rho) = 0.00208 \) m\(^2\)/kg. Consequently, we have

\[
K = 2.69 \times 10^3 \text{ J/m}^2 \times 0.00215 \text{ m}^2/\text{kg}
\]

\[
= 5.79 \text{ J/kg} = 5.79 \text{ Gy}
\]

\[
K_{coll} = 2.69 \times 10^3 \text{ J/m}^2 \times 0.00208 \text{ m}^2/\text{kg}
\]

\[
= 5.60 \text{ J/kg} = 5.60 \text{ Gy}
\]

\[
K_{rad} = K - K_{coll}
\]

\[
= 5.79 - 5.60 \text{ Gy}
\]

\[
= 0.19 \text{ Gy}
\]

(b) Approximately what is the absorbed dose in the foil, assuming no charged particles are incident from elsewhere?

Because the foil is of negligible thickness, we really do not know what has happened to the secondary electrons, so the dose is indeterminate.

(c) What would happen to \( K, K_{coll}, K_{rad} \) and \( D \) if a strong magnetic field were applied with lines of force lying in the field?

If the magnetic field is sufficiently strong so that the secondary electrons are not able to escape from the foil, then the value of the dose will approach that of the collision kerma.

(d) What is the value of \( g \)?

\[
g = \frac{(\mu_{tr}/\rho) - (\mu_{en}/\rho)}{(\mu_{tr}/\rho)}
\]

\[
= \frac{0.00215 - 0.00208}{0.00215}
\]
5. (Attix 4-7) A broad beam of low-energy x-rays with a fluence rate of $3.7 \times 10^{-4} \text{ J/cm}^2 \text{ s}$ irradiates a plate of Al perpendicularly, and is completely absorbed.

(a) What is the energy absorbed per cm$^2$ in 5 min?

Assuming all the energy is absorbed by the Al plate, the energy absorbed is equal to the energy incident in 5 min, or

$$E = 3.7 \times 10^{-4} \text{ J/cm}^2 \text{ s} \times 5 \text{ min} \times 60 \text{ s/min}$$

$$= 0.111 \text{ J}$$

(b) If the slab is 2 cm thick and has a density of 2.7 g/cm$^3$, what is the average value of $K_{coll}$ throughout the medium?

$$K_{coll} = \frac{0.111 \text{ J}}{\text{cm}^2} \times \frac{1}{2 \text{ cm}} \times \frac{\text{cm}^3}{2.7 \text{ g}} \times \frac{1000 \text{ g}}{\text{kg}}$$

$$= 20.6 \text{ J kg}^{-1}$$

$$= 20.6 \text{ Gy}$$

(c) Assuming no electrons enter or leave the plate, what is the average absorbed dose?

Because no electrons enter or leave the plate, we have charged particle equilibrium, and $K_{coll}$ is equal to $D$, so the absorbed dose is also 20.6 Gy.

(d) What would be the average absorbed dose if the slab were 4 cm thick?

If the slab were twice as thick, then the volume would be doubled, and the dose would be halved, so the dose would be 10.3 Gy.