Lecture 3.1e/f Unclear Points:

What is the "relative probability per energy interval of generating a positron with" some energy (Slide 12)? I don't understand the implication of the "relative probability," which often exceeds 1.

The graph (taken from J&C) indicates relative probability per fractional energy interval. The integral under the curve is unity, since the relative probability is 1.0.

For #1 in pre-test, I was kind of confused because the cross section for pair production, from the equation, is proportional to $Z^2$, but the mass attenuation coefficient is proportional to $Z$. I chose "proportional to $Z$" because I thought the mass attenuation coefficient is the more closely related, but why do the cross section and the mass att. coeff. have different $Z$ dependence?

\[ \frac{d\kappa}{d\Omega} \] is an atomic attenuation coefficient. To get the mass attenuation coefficient we divide by $Z$.

How does pair cross section relate to pair production? How does pair production attenuation coefficient relate to pair production? How does pair cross section relate to pair production attenuation? When we say "pair production is dependent on $Z^2$", are we looking at pair cross section or pair production attenuation? (Side note: I believe pair production attenuation coefficient gives us the probability of interaction, and that is what we look at when try to find a dependence on pair production, but I am uncertain.)

Cross section is attenuation coefficient. Probability of interaction per unit atomic density.

At the beginning of the lecture you say that the pair cross section is dependent on $Z^2$ (slide 9), and then towards the end you say that it is actually dependent on $Z$ (slide 26). Which is it?

See explanation above.

If a high energy photon can spontaneously result in pair production in a vacuum, why are we only considering this process happening in the vicinity of a nucleus?

Pair production cannot occur in free space. Need Coulomb field of either nucleus or electron to conserve momentum.

For question 1, isn't the answer $Z$ or $Z^2$ depending on if you are referring to pair production per atom ($Z^2$) or per unit mass ($Z$)?

Yes.

Energy distribution formula in slide 5 and slide 14 seems not consistent.

They are not. Remove the rightmost part of the equation.

Other than PET, are there any imaging or therapy applications with pair production?
PET is not really related to pair production, but rather to positron emission. Major concern with therapy application is that attenuation coefficient increases with increasing energy. Consequently penetrating ability of radiation decreases with increasing energy. This, coupled with increased neutron production, places practical upper limit on photon energy used for radiation therapy.

According to Cunningham’s text, the high energy photon is more easily stopped in a pair process than a lower energy photon, making the high energy photon less penetrating than the low energy photon. Why is that?

I have not found a good qualitative explanation why the probability of pair production increases with increasing photon energy.

I’m still confused over the dependence of pair production on Z. In question 1, are you talking about the pair coefficient per atom or per unit mass? Because one depends on Z and the other depends on Z^2.

The answer is Z or Z^2 depending on if you are referring to pair production per atom (Z^2) or per unit mass (Z). The question was poorly worded.

Why do we need 2.04 MeV for triplet production? I understand you need 1.022 MeV for pair production. But I don’t know how the rest 1.022 MeV comes from the conservation of momentum and energy.

In order to conserve energy and momentum the threshold energy for both pair and triplet production is

\[(hv)_{\text{min}} = 2m_0c^2 \left( 1 + \frac{2m_0c^2}{Mc^2} \right)\]

If \(M=M_{\text{nuc}}>>m_0\), we have pair production, but if \(M=m_0\), we have triplet production.

Is Pair Production near the nucleus more prevalent than Triplet Production near electrons because of the greater strength of the electric field near the nucleus?

The triplet cross-section is approximately 1/Z times the pair cross-section. Again, I could not find a good qualitative answer why this is so.

Can you explain the meaning of the graph of the energy distribution spectrum for pair production - namely, what do we mean when we say the energy distribution is “flat”? What does this physically mean?

I assume you are referring to slide 19 in which we look at how the energy is distributed between the positron and the electron. When we say that the distribution is “flat,” we mean that the
relative probability per energy interval of generating a positron with a given energy is relatively independent of how energy is distributed between the positron and the electron.

What is meant by an electron undergoing a transition from a negative energy state to a positive energy state? Is the electron, staying within the same orbit, but emitting a positron, and thus undergoing a "transition" into a positive energy state?

Compare the Feynman diagrams for pair production and Bremsstrahlung:

![Feynman diagrams](image)

Also, on slide 20, where does the fractional energy of 0.2 MeV come from?

The graph on slide 19 is relative probability per energy interval of generating a positron with a given energy, so to get relative probability we must multiply the quantity on the graph by the energy interval.

Maybe I just missed this earlier - but what do you mean when you say "photoelectric effect is a type 3a process"?

See slide 3 of the lecture on photoelectric effect, which categorizes the photon interactions with matter.
Dumb question - in the pair prod. cross section equation on page 8, that D-omega in the denominator, that is the solid angle bit, like we saw previously, right?

No dumb questions, but you are right.

On page 19 on the graph for "relative probability", that graph isn't normalized to 1.0, so you get the value of 1.09. why isn't that normalized to 1.0, and they why did that work correctly in the example problem.

Area under the curve is 1.0. The value of 1.09 is the relative probability per energy interval of generating a positron with a given energy, so to get relative probability we must multiply the quantity on the graph by the energy interval.

My curiosity has the better of me: If the probability of pair production increases with nuclear mass (2Z is mentioned), then I am tempted to think this is a nuclear interaction or some kind of creepy field interaction with an electron in a "negative energy state". I'm sure this is far beyond anything we understand for our class (amen), but then later in the lecture don't you say sometimes pair production happens just in the vicinity of an electron without a nucleus being around (if I got that right). Plus the fine structure constant being involved - I did not see THAT coming. So if there was a "simple" intuitive explanation for pair production I feel sure you would have given it to us, and I don't want to dwell on it. The Negative energy state of an electron - is that a real thing? If pair production was only coulombic interaction, wouldn't we expect only to see the planck constant and not the fine structure? (sorry this was so long). Fine structure constant has no units, smells like a fudge factor. Feel free to ignore this for class.

Pair production is an interaction that involves a nucleus in order to conserve momentum.

Triplet production is an interaction that involves an electron in order to conserve momentum.

You say in the lecture, "In pair production the electron can be looked at as undergoing a transition from a negative energy state, creating a positron, to a positive energy state, absorbing a photon." What does it mean for an electron to be in a negative energy state? Is it the electron being in a negative energy
state that creates the positron, or is it the electron transitioning out of the negative energy state that creates the positron? And aren’t all electron energies technically negative?

In the quantum-mechanical model using the Dirac equation, electrons populate the “negative energy” solutions to the Dirac equation. According to the Dirac equation, an electron localized in space will have components of its wave function which are “negative energy”. The more localized the state, the greater the “negative energy” content. Electrons, making transitions from a negative energy state to a positive energy state, absorb a photon equal in energy to $2m_0c^2$.

An alternative formulation of quantum field theory looks at the “negative energy” solution as a positive energy solution moving backward in time.

In question 2, you ask for the dependence of pair production on Z. Are you referring to the mass or atomic attenuation coefficient?

The answer is $Z$ or $Z^2$ depending on if you are referring to pair production per atom ($Z^2$) or per unit mass ($Z$). The question was poorly worded.

Can you explain problem 3? I chose "the annihilation photons have good Z contrast". My reasoning for this selection was that pair production is typically outside of the imaging energy range, since it becomes important at energies greater than about 30 MeV. However I am uncertain of this answer as I know that pair production is important in PET imaging and that it is proportional to $Z$.

Pair production is not important in PET imaging; positron emission is the relevant process.

In the expression for differential scattering cross section for pair production, you say that the constant “137” is the “fine structure constant” from Quantum mechanics. Please explain what it is and it’s role in the expression.

The fine structure constant is a dimensionless number, equal to $1/137$, that characterizes the strength of the electromagnetic interaction between elementary charged particles.

I didn’t understand the variation in the probability of energy distribution at low (5Mev) and high (20Mev) energies. Could you please explain?

Not clear why the energy split goes from 50-50 at low energies to around 25-75 at high energies. J&C cite Evans, who cites a 1952 Rev Mod Phys paper by Davisson and Evans.