Mathematics of Radioactive Decay

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Purpose

• To define and present methods for calculating parameters and quantities that describe radioactive decay

Let's look at another patient

• 57 yr old male
• Stage T2a adenocarcinoma of prostate
• Lives far from MDACC so elected implantation of radioactive sources
**$^{125}$I implant**

- Why did we use $^{125}$I?
- What type of radiation is emitted by $^{125}$I?
- What is the mechanism by which the radiation is emitted?
- How does this radiation differ from external beam radiation?

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**Quantities related to decay process**

- How much radioactive material is present?
- How much radiation is produced?
- How do these quantities change with time?

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**How much radioactive material is present?**

- Activity – number of disintegrations per unit time
  - SI unit:
    - 1 Becquerel (Bq) = 1 disintegration/sec
  - Commonly used unit:
    - 1 Curie (Ci) = $3.7 \times 10^{10}$ disintegrations/sec
  - Originally defined as decay rate of 1 g of radium
Definition of activity

- Multiples of Curie:
  1 millicurie (mCi) = $3.7 \times 10^7$ dps
  1 microcurie (μCi) = $3.7 \times 10^4$ dps
  1 nanocurie (nCi) = $3.7 \times 10^1$ dps
  1 kilocurie (kCi) = $3.7 \times 10^{13}$ dps

Definition of activity

- Activity is proportional to the number of decaying atoms present
  \[ A = \lambda N \]
  where \( \lambda \) is the transformation constant

Definition of activity

- At any given time:
  \[ N = N_0 e^{-\lambda t} \]
  \[ A = A_0 e^{-\lambda t} \]
Definition of half life

- Half life – that time for $N$ to equal $\frac{1}{2}N_0$
  - or that time for $A$ to equal $\frac{1}{2}A_0$
  - Relate to transformation constant by
    $$T_{1/2} = \frac{0.693}{\lambda}$$
- Generally, half life is tabulated, rather than transformation constant

Definition of average life

- Suppose we had hypothetical radionuclide with an initial activity equal to that of our real nuclide. If its decay were uniform rather than exponential, how long would it take to decay completely?
  $$T_{avg} = 1.44 T_{1/2}$$

Application of average life

- Suppose we have 1 $\mu$Ci $^{131}$I ($T_{1/2} = 8.0$ d).
  How many disintegrations will we have until complete decay?
  $$1 \mu\text{Ci} = 3.7 \times 10^4 \text{ dps}$$
  $$T_{avg} = 1.44 \times 8.0 \text{ d} \times 24 \text{ h/d} \times 60 \text{ m/h} \times 60 \text{ s/m}$$
  $$T_{avg} = 9.95 \times 10^5 \text{ s}$$
Application of average life

• Suppose we have 1 μCi $^{131}$I ($T_{1/2} = 8.0$ d). How many disintegrations will we have until complete decay?

$T_{avg} = 9.95 \times 10^5$ s

$N = 3.7 \times 10^4$ dps $\times 9.95 \times 10^5$ s

$= 3.68 \times 10^{10}$ disintegrations

Transient equilibrium

• Often radioactive parent decays to radioactive daughter

• Radioactive daughter may have shorter half life than parent
  – Daughter produced by parent as rapidly as it decays
  – Daughter then decays with apparent half life of parent

Example:

$^{132}$Te ($T_{1/2}=78$ h) $\rightarrow$ $^{132}$I ($T_{1/2}=2.3$ h)
Transient equilibrium

- After equilibrium is reached:

\[
\frac{A_d}{A_p} > \frac{A_p}{\lambda_d - \lambda_p}
\]

Application of transient equ

Production of $^{99m}\text{Tc}$ (half life = 6.0 h) from $^{99}\text{Mo}$ (half life = 66 h) in radionuclide generator

Activity of daughter less than that of parent because not all parent decays to daughter

Secular equilibrium

- Half life of parent >> Half life of daughter
- Activity of daughter equals that of parent
Secular equilibrium

Example:

\[ ^{226}\text{Ra} \ (T_{1/2}=1620 \text{ y}) \rightarrow ^{222}\text{Rn} \ (T_{1/2}=3.83 \text{ d}) \]

Decay series

- Long lived parent → Decay products → Stable daughter
- Radioactive daughter products in secular equilibrium with parent
- Apparent half life for decay of daughter same as that of parent

Decay series

- \(4n+2\) series:
  \[ ^{238}\text{U} \rightarrow \ldots \rightarrow ^{206}\text{Pb} \]
  \[ ^{226}\text{Ra} \text{ is important daughter of series} \]

- \[ ^{226}\text{Ra} \ (T_{1/2}=1620 \text{ y}) \rightarrow ^{222}\text{Rn} \ (T_{1/2}=3.83 \text{ d}) \]
Decay series

• 4n+3 series:
  $^{235}\text{U} \rightarrow \ldots \rightarrow ^{207}\text{Pb}$
• 4n series:
  $^{232}\text{Th} \rightarrow \ldots \rightarrow ^{208}\text{Pb}$
• 4n+1 series:
  $^{241}\text{Am} \rightarrow \ldots \rightarrow ^{209}\text{Bi}$

Decay series

• 4n+1 series:
  $^{241}\text{Am} \rightarrow \ldots \rightarrow ^{209}\text{Bi}$
  - Does not exist in nature since parent is not long lived
• Decay is generally via α or β decay.